

A Rigorous Comparison of the D-Wave 2X QPU to Established B-QP Solution Methods

Carleton Coffrin, Harsha Nagarajan, Russell Bent

A-1 & T-5

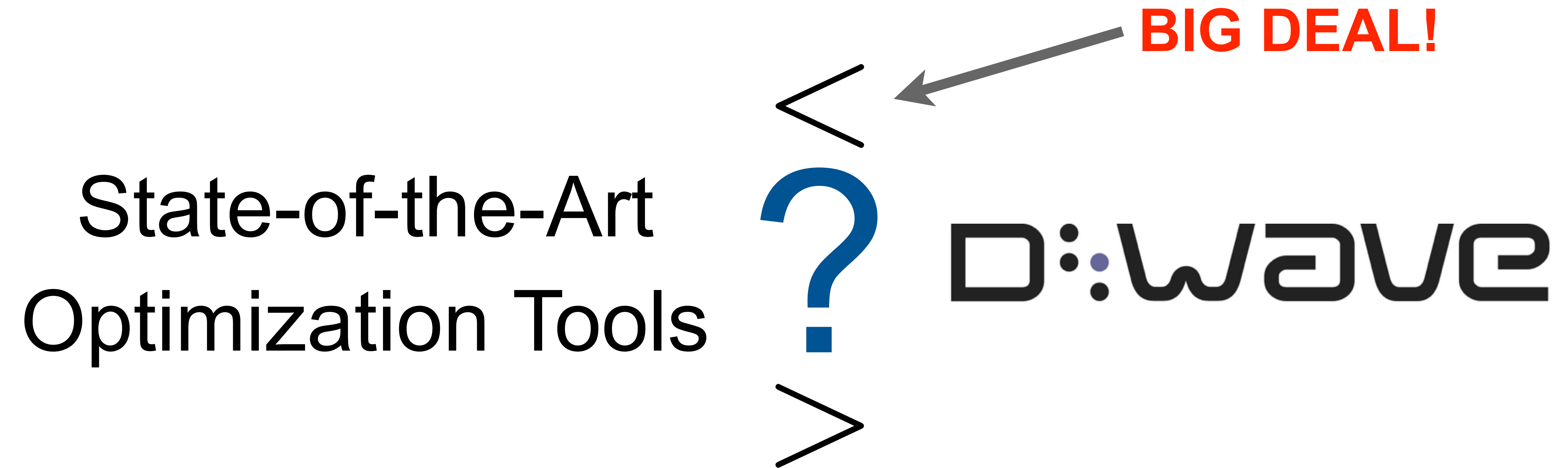
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D-Wave for Optimization

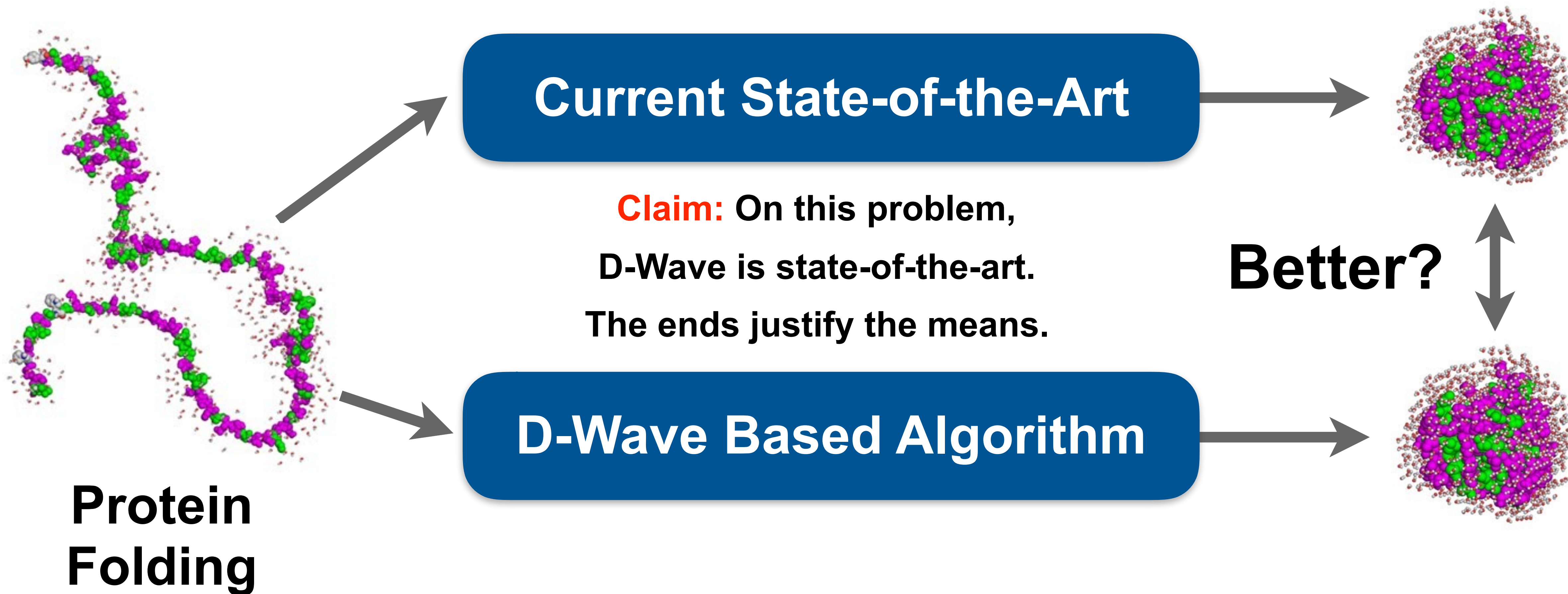
Binary-Quadratic Program
(B-QP / QUBO)

NP-Hard Combinatorial Optimization Problem

The Benchmarking Question




A Preferable Benchmarking Situation



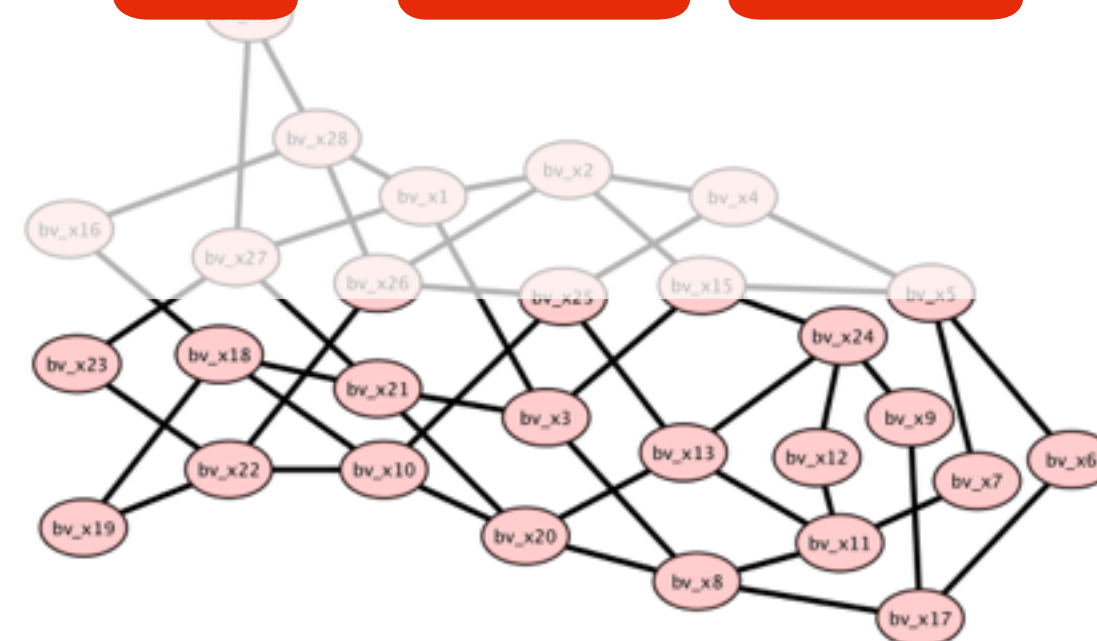
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The Benchmarking Problem

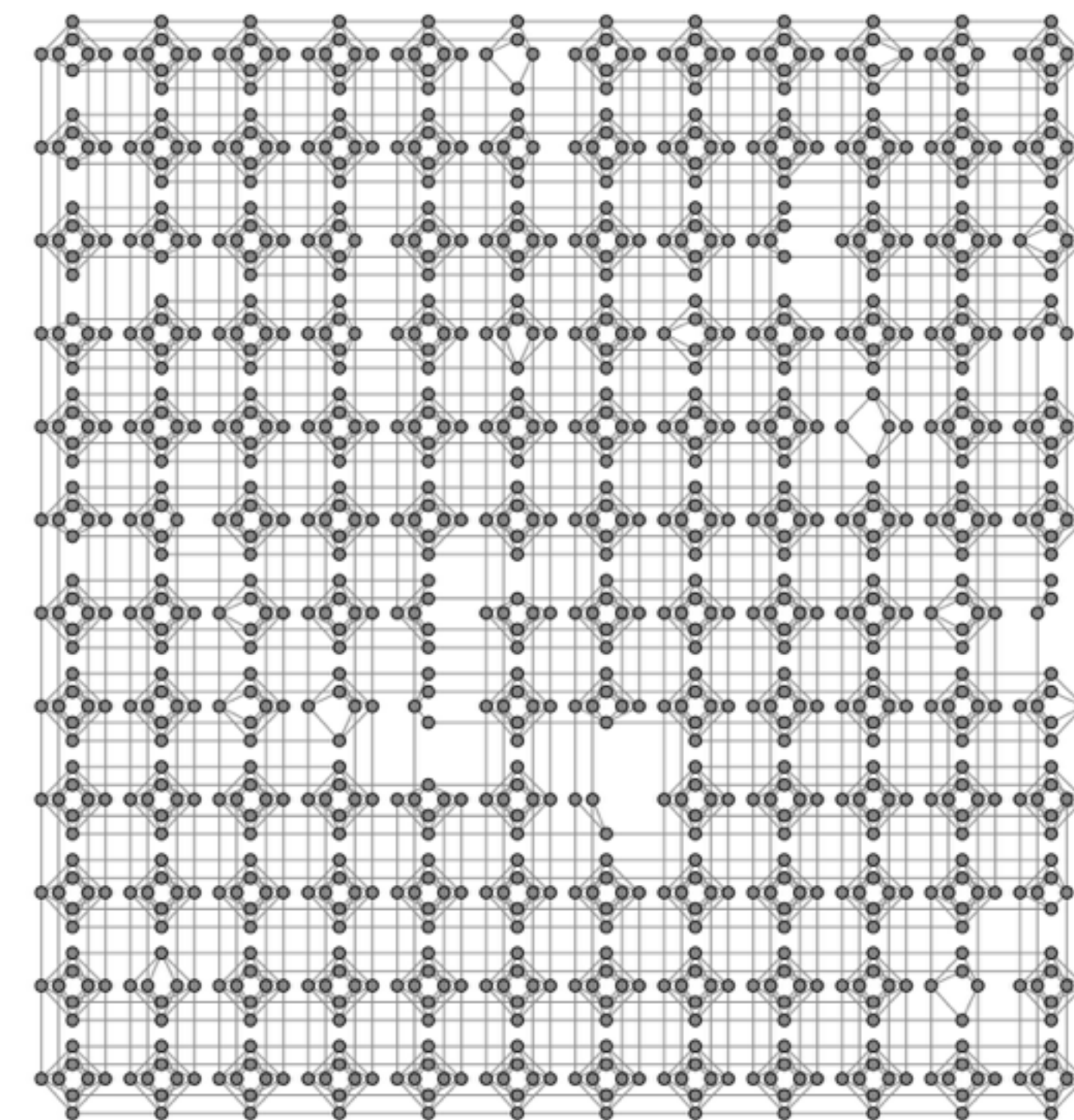
ISTI '16


 QPLIB
 DIMACS
 Max-Clique

FAIL



D:wave

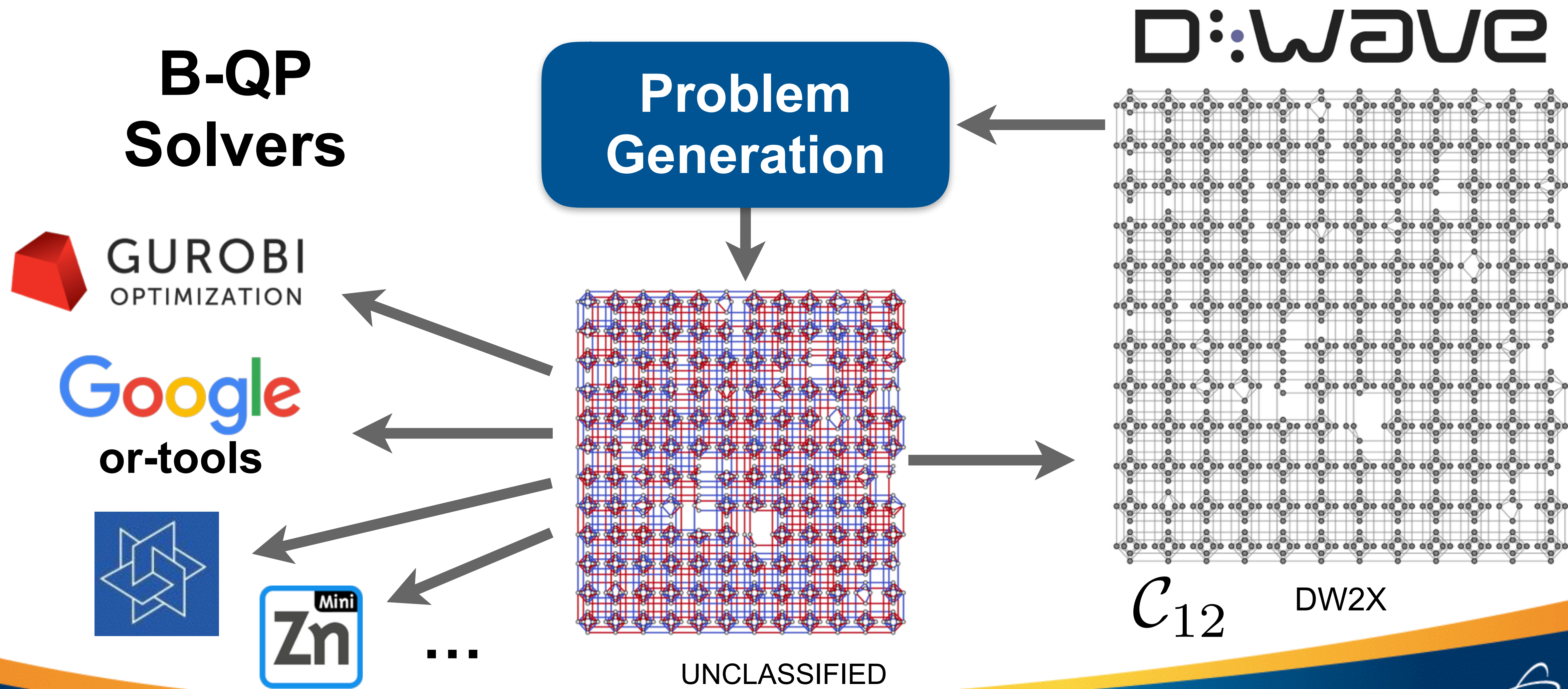


C_{12}

DW2X

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A Benchmarking Stopgap



Problems with Problem Generation

- How hard are randomly generated problems?
- Lessons learned from **Random SAT**

Hard and Easy Distributions of SAT Problems

David Mitchell

Dept. of Computing Science
Simon Fraser University

Bart Selman

AT&T Bell Laboratories
Murray Hill, NJ 07974

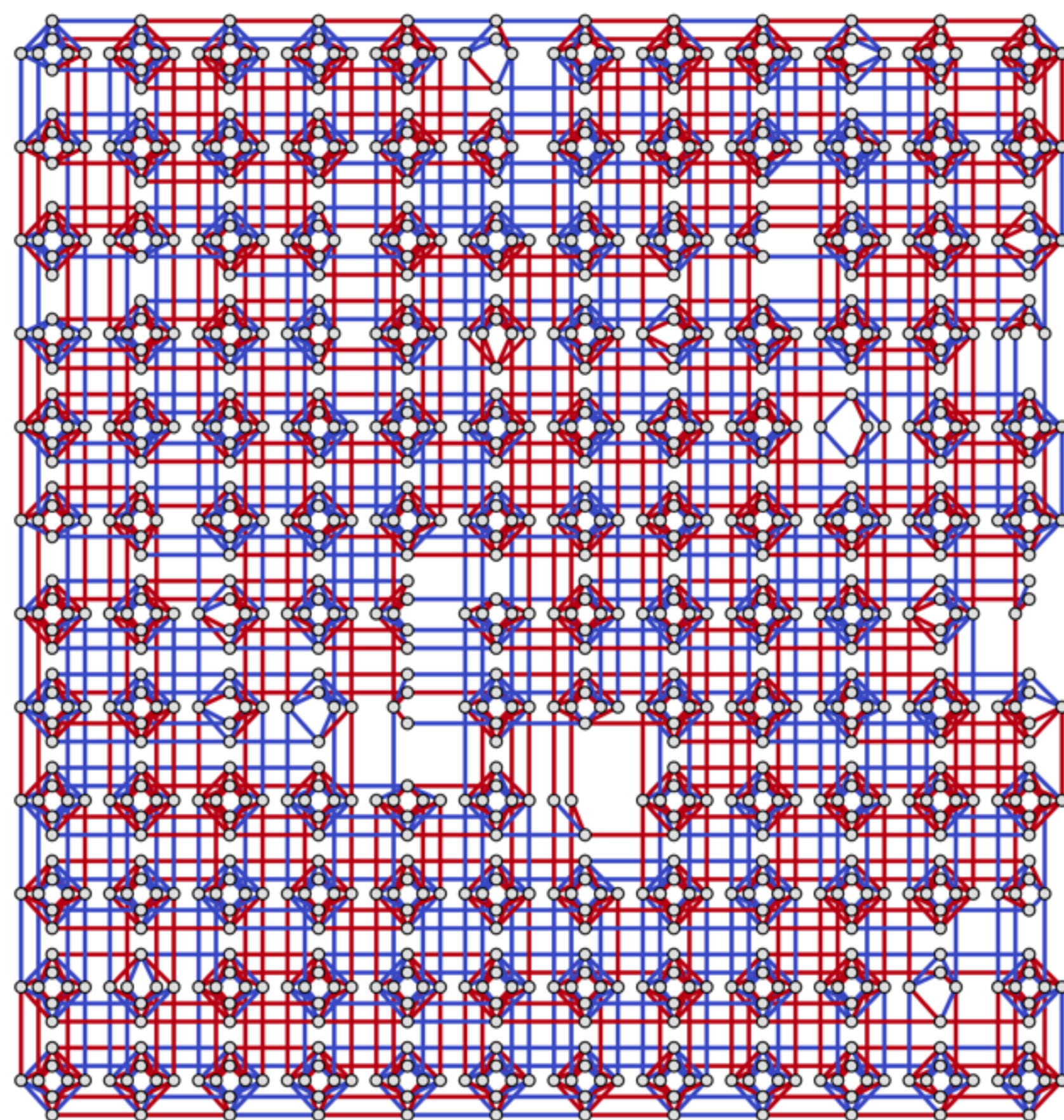
Hector Levesque*

Dept. of Computer Science
University of Toronto

AAAI-92

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Example: What's the Difference?

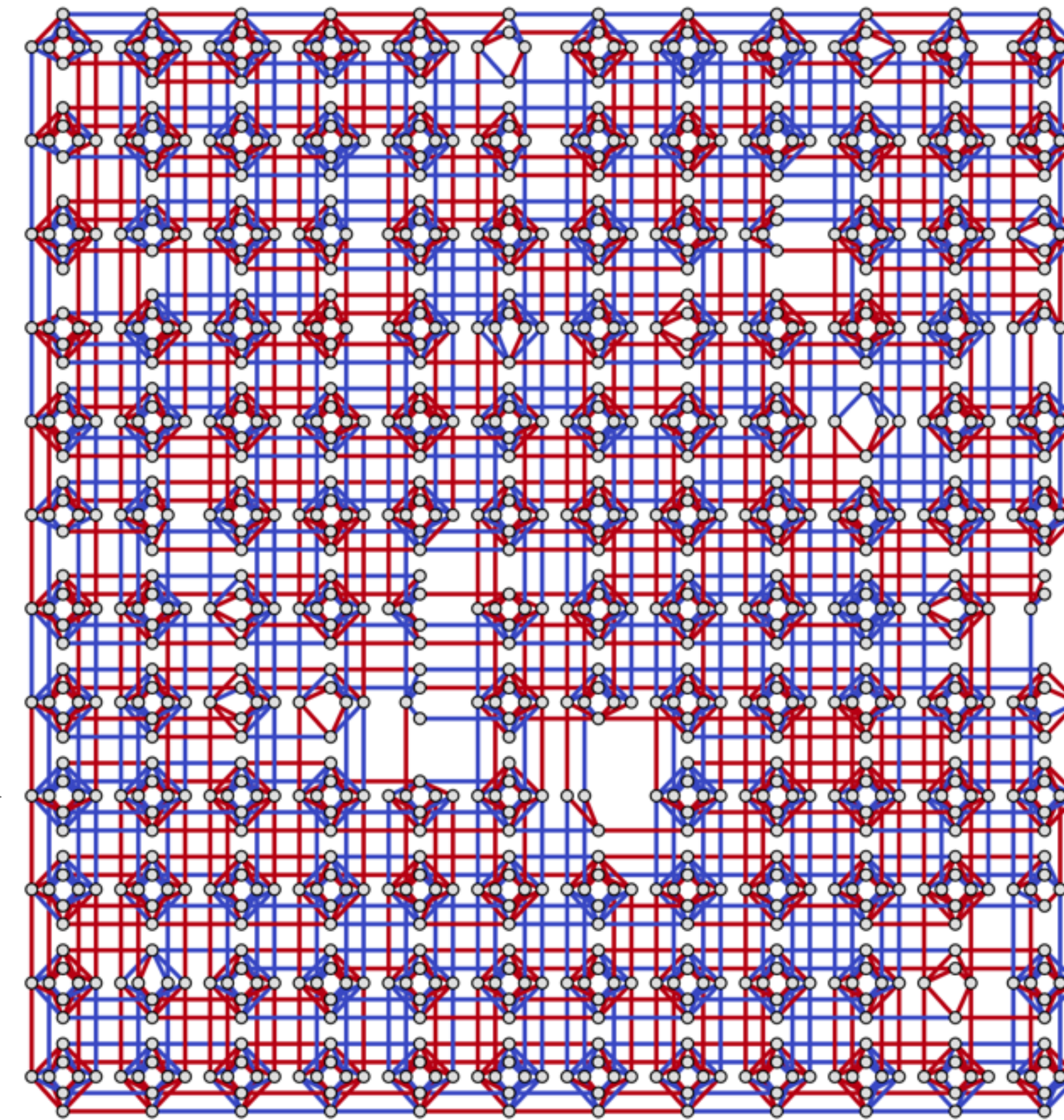


HARDEST

←
super frustrated
system

EASIEST

→
ferromagnet
in disguise!



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The Key Challenge

How to generate a **HARD** D-Wave case



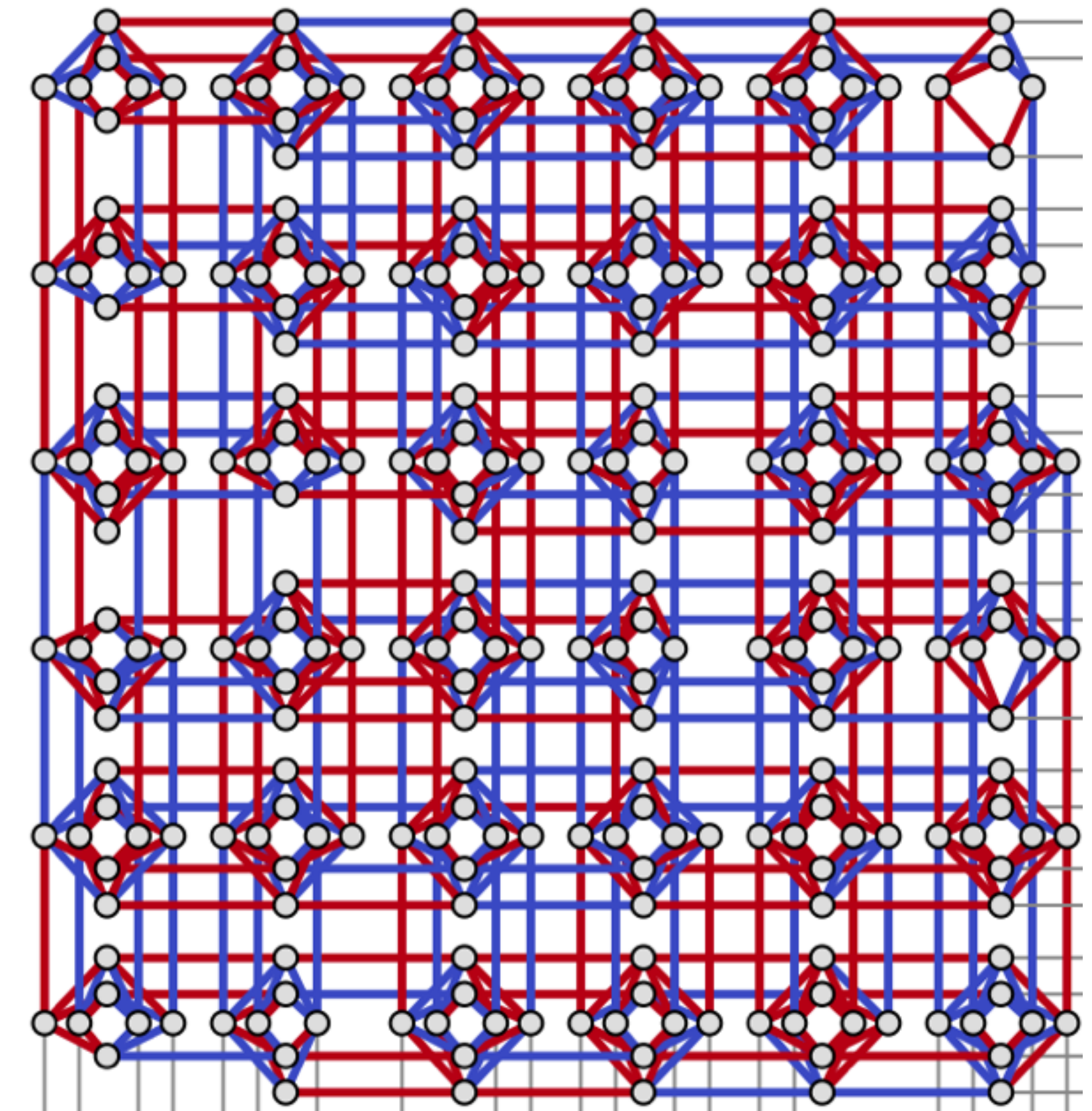
look to the literature

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Proposed Problem Generators

- **RAN-k**
 - set field to zero
 - set couplers at random
 - use k discrete steps

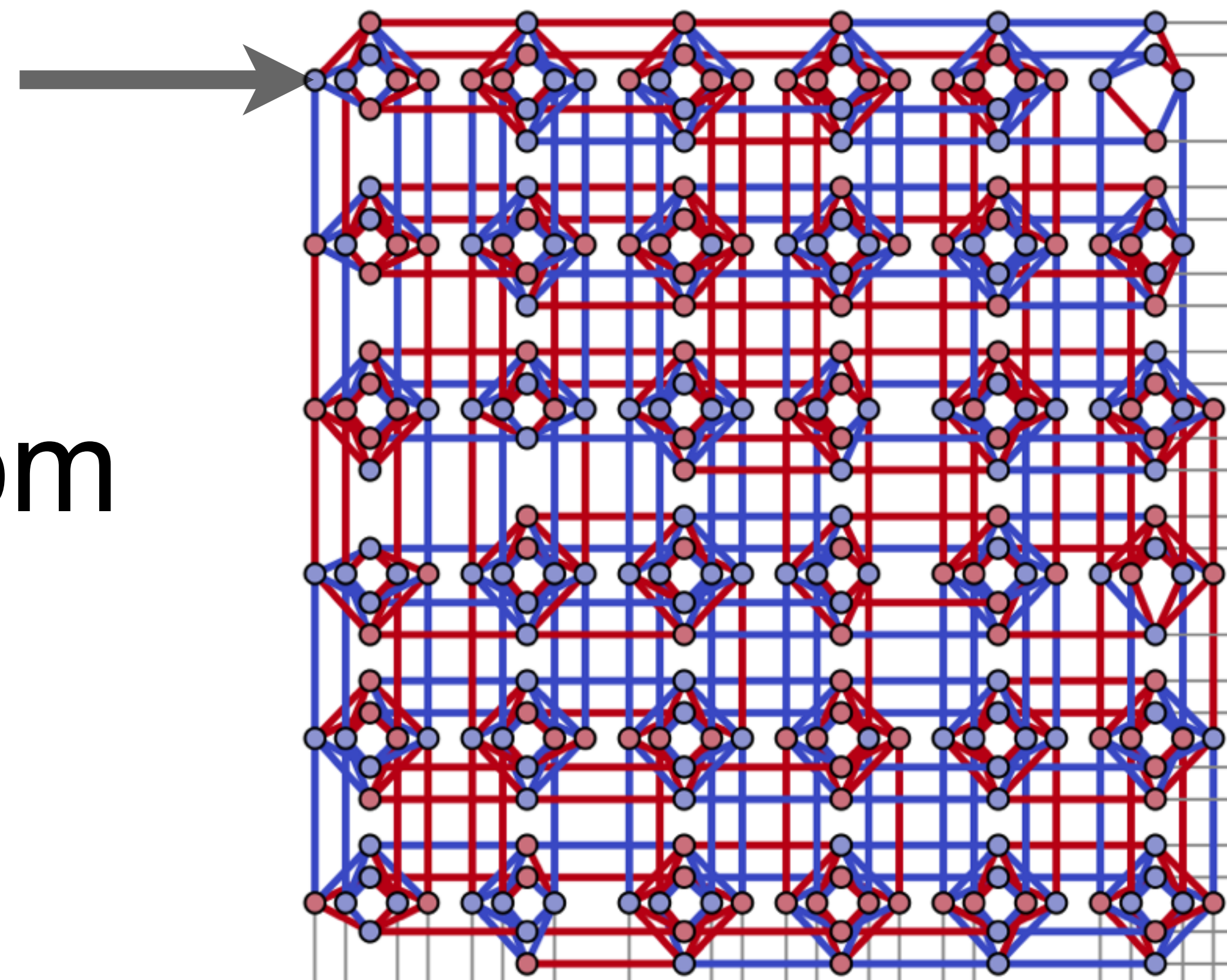
<https://arxiv.org/abs/1508.05087>



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Proposed Problem Generators

- **RANF-k**
 - set field at random
 - set couplers at random
 - use k discrete steps

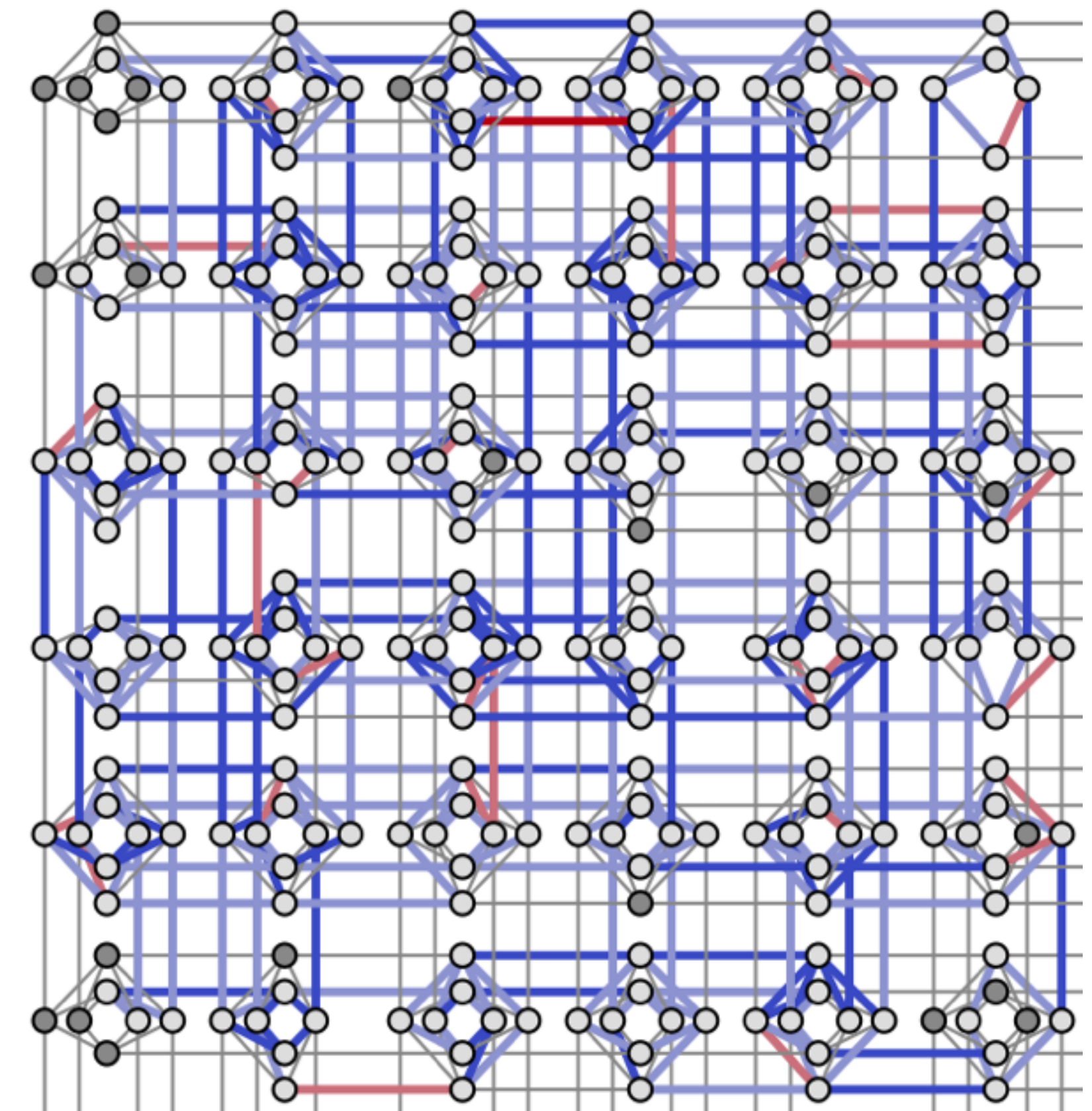


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Proposed Problem Generators

- Frustrated Loops (FL)
- find random cycles
- add one edge of frustration
- overlay multiple cycles

<https://arxiv.org/abs/1508.05087>



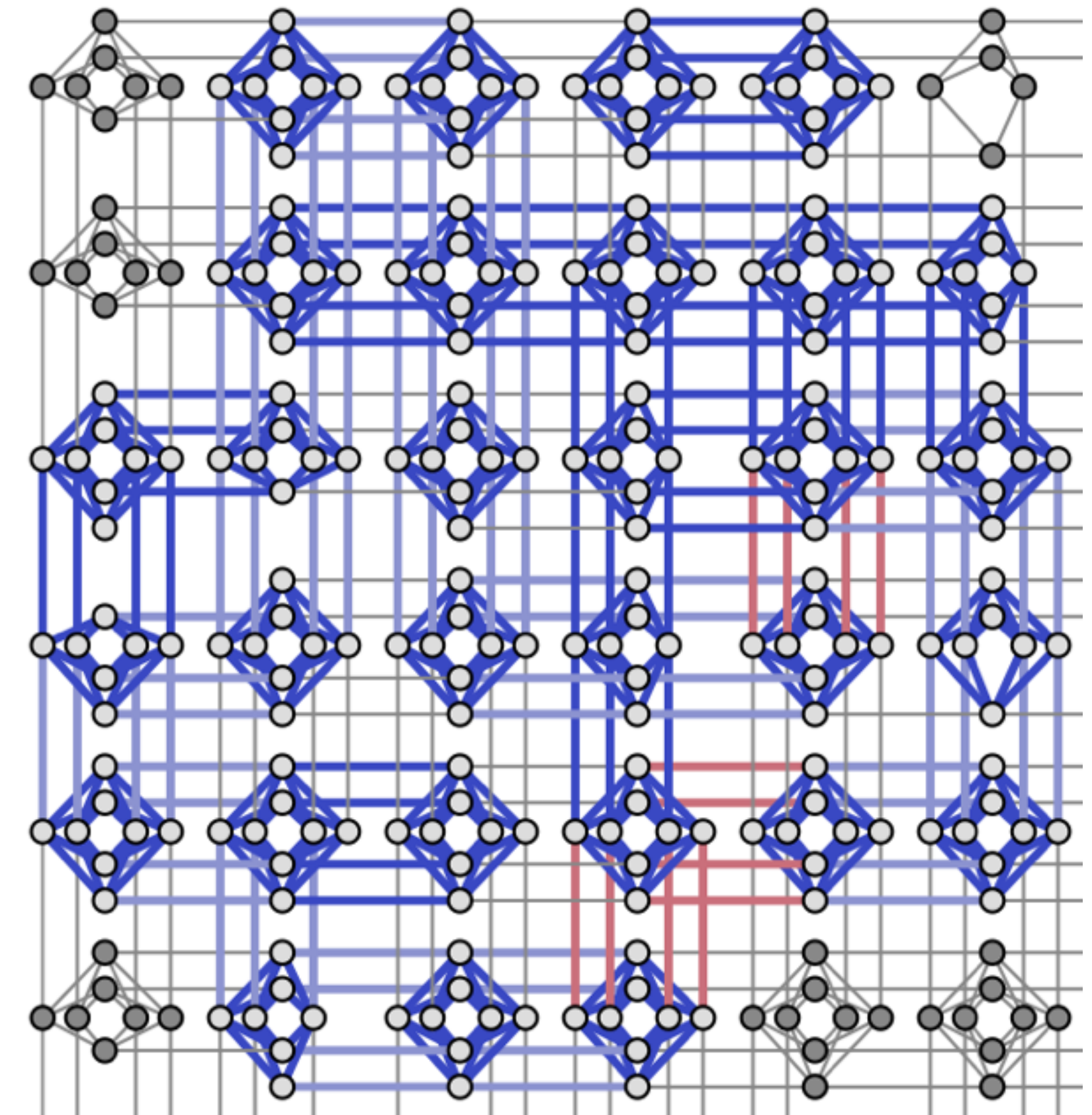
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Proposed Problem Generators

- Frustrated Cluster Loops (FCL)
- find random cycles
- add one edge of frustration
- overlay multiple cycles

<https://arxiv.org/abs/1701.04579>

<https://arxiv.org/abs/1703.00622>

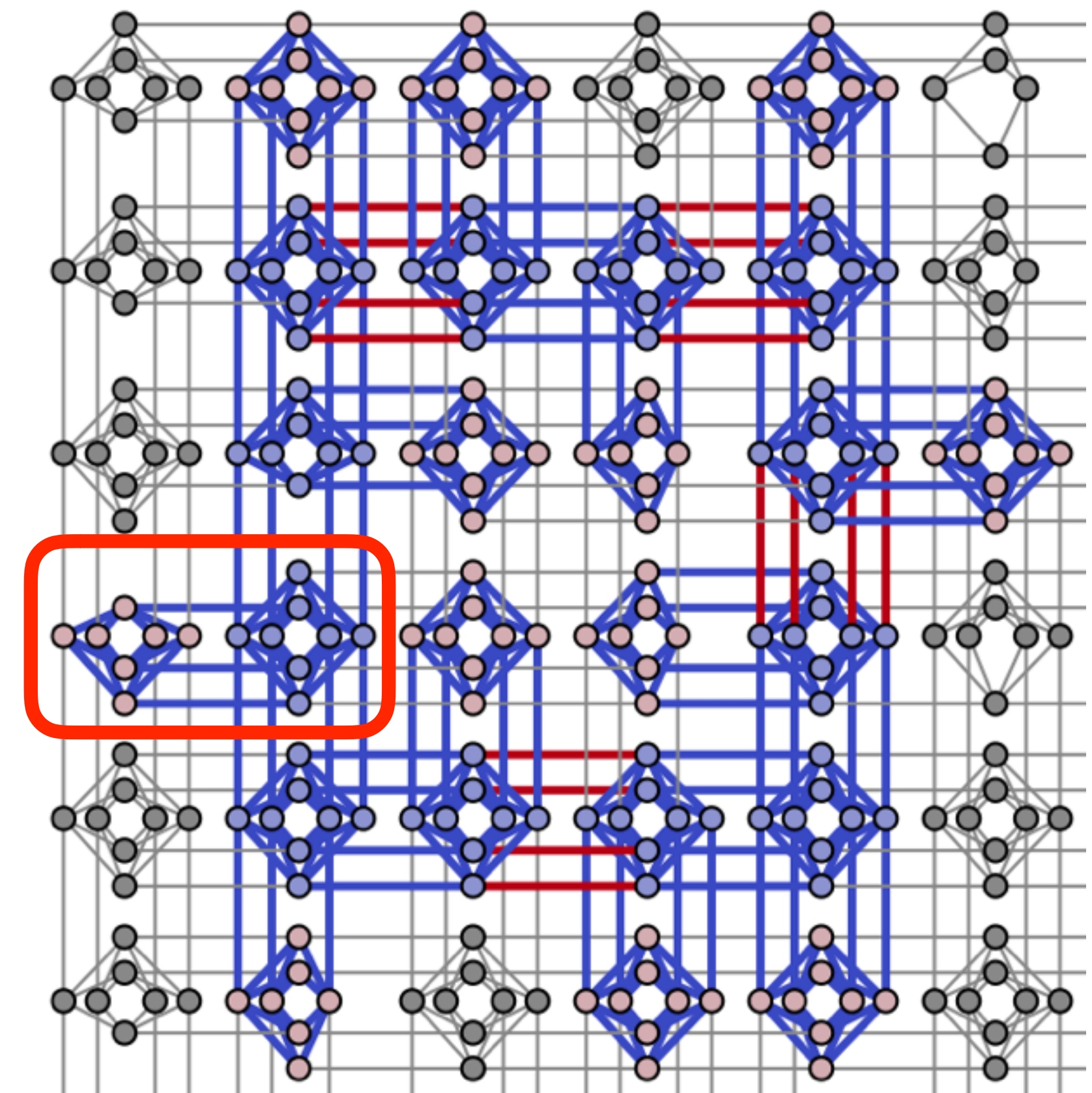


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Proposed Problem Generators

- Weak-Strong Cluster Networks (**WSCN**)
- random grid of two cell gadgets

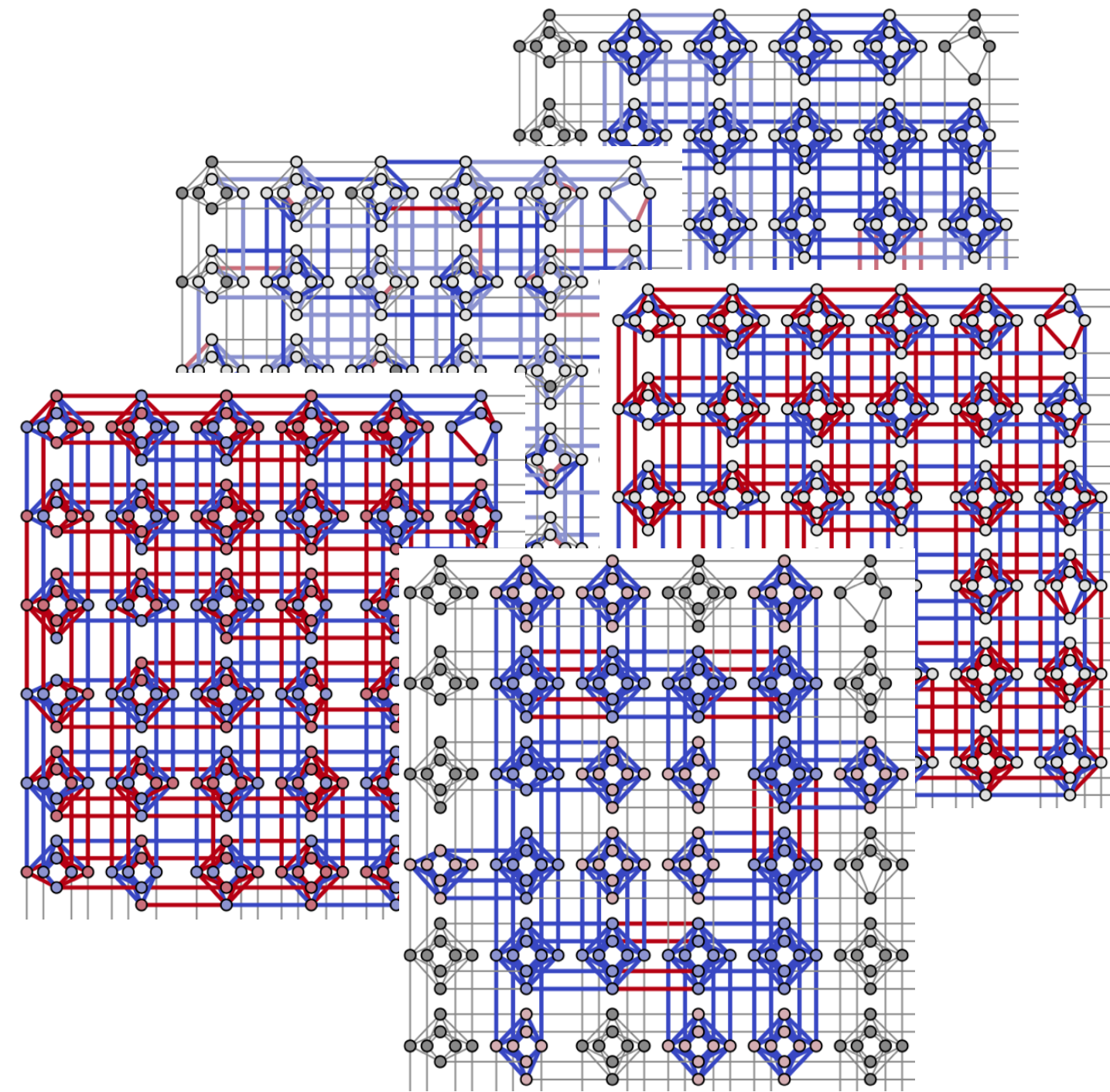
<https://arxiv.org/abs/1512.02206>



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What to Compare?

Cases



?



Solvers

qbsolv



D:wave



GUROBI
OPTIMIZATION

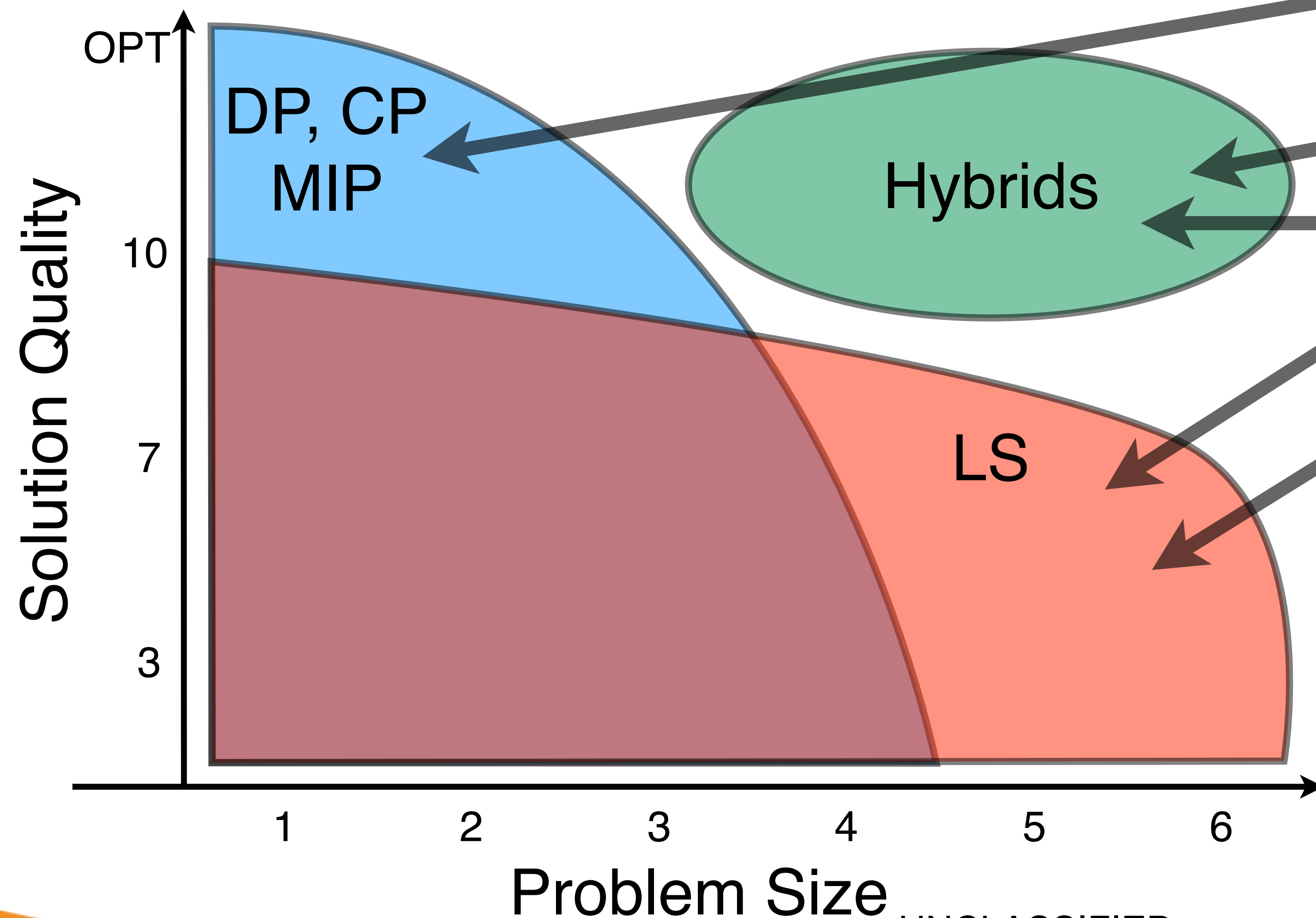
HFS

SA

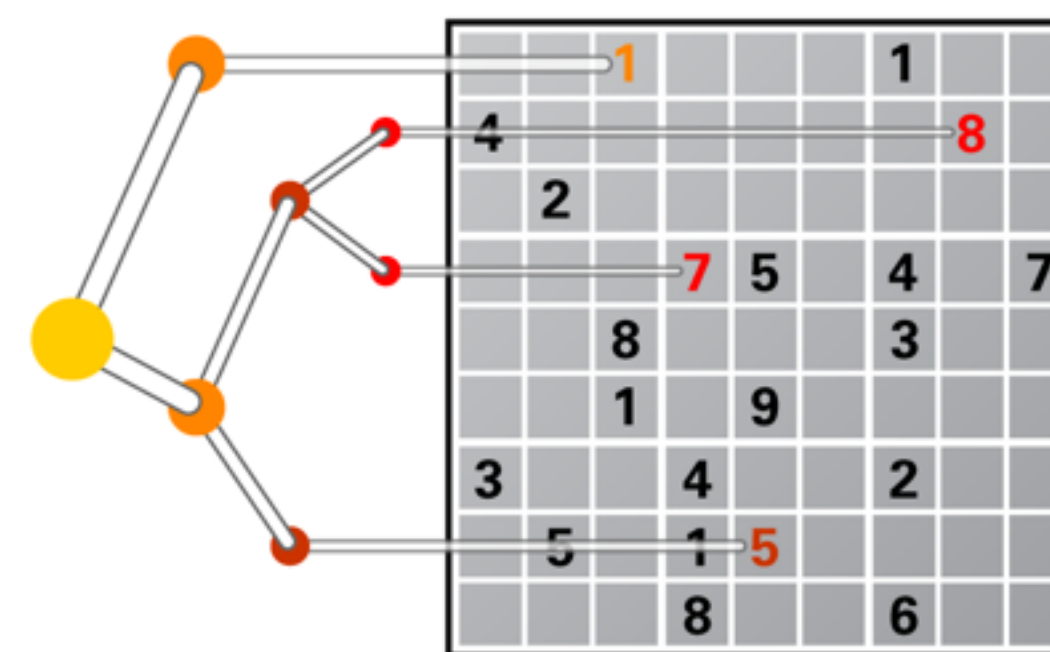


Google
or-tools

Types of Solvers



coursera



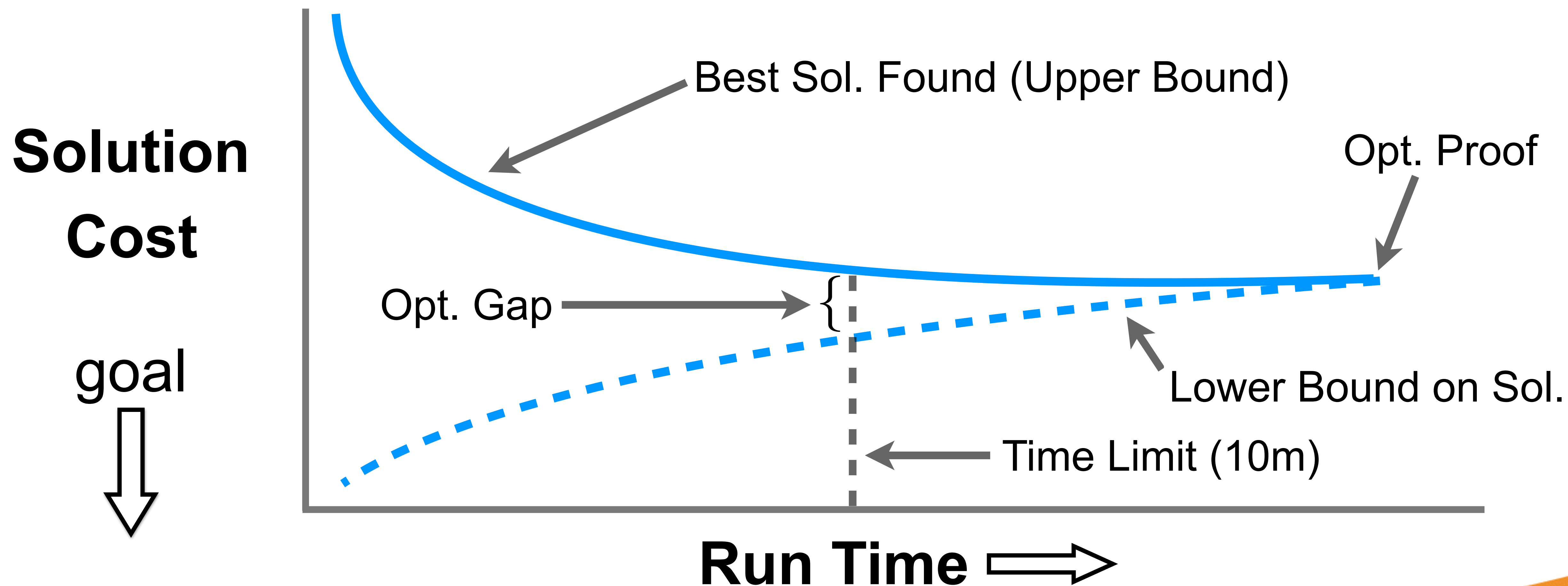
Discrete Optimization

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Classic B-QP Solver



GUROBI
OPTIMIZATION



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HFS Solver

Hamze-de Freitas-Selby (HFS)

<https://arxiv.org/pdf/1207.4149.pdf>

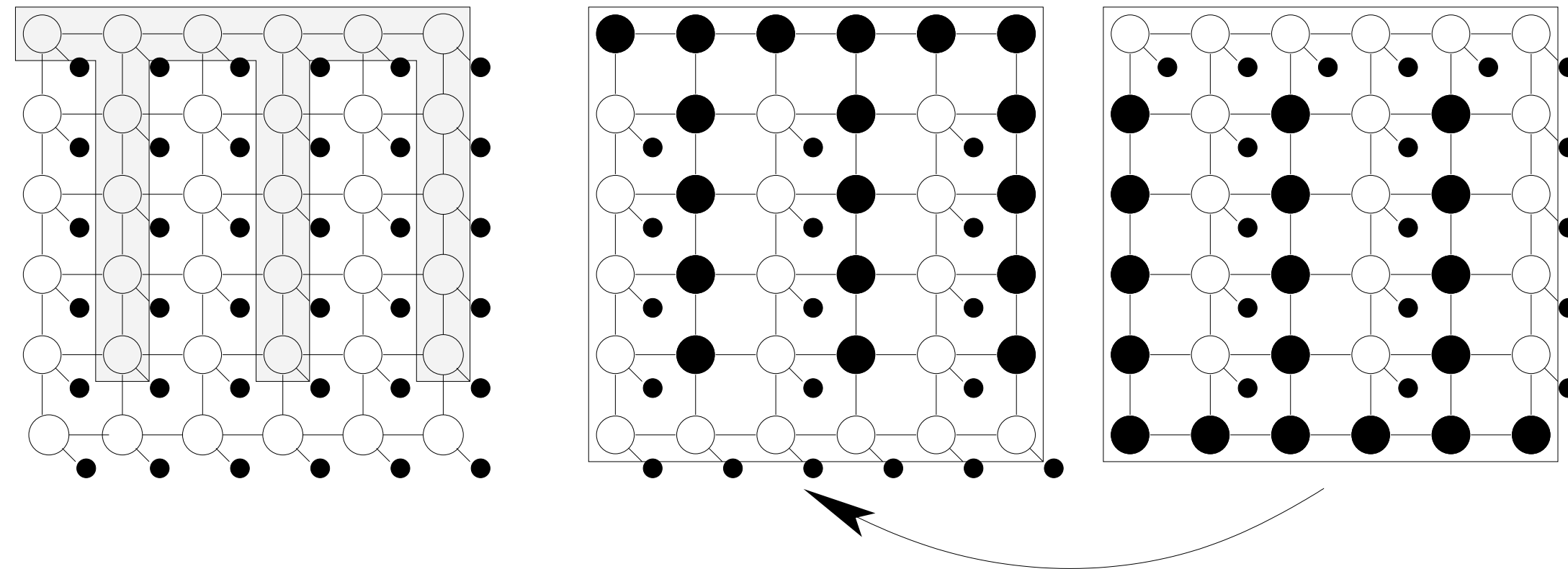
<https://github.com/alex1770/QUBO-Chimera>

HF
S

Low Treewidth

Subgraphs

Optimization Loop using DP

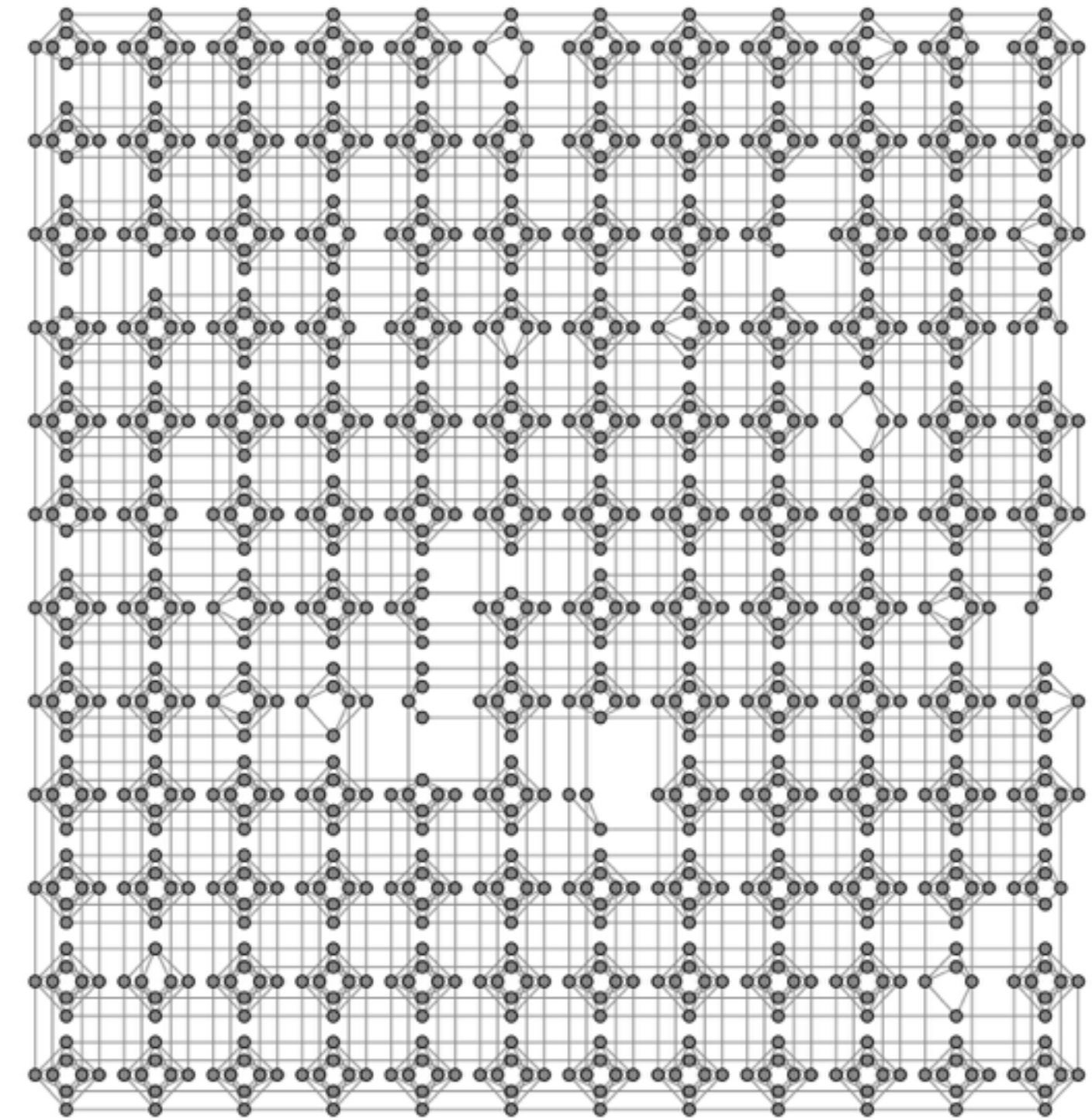


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D-Wave For Optimization

- 10,000 samples @ 5 micro seconds each
- random gauge transform every 100 samples
- use the **best** of all samples
- Takes 3.5 seconds on the QPU

D:WAVE

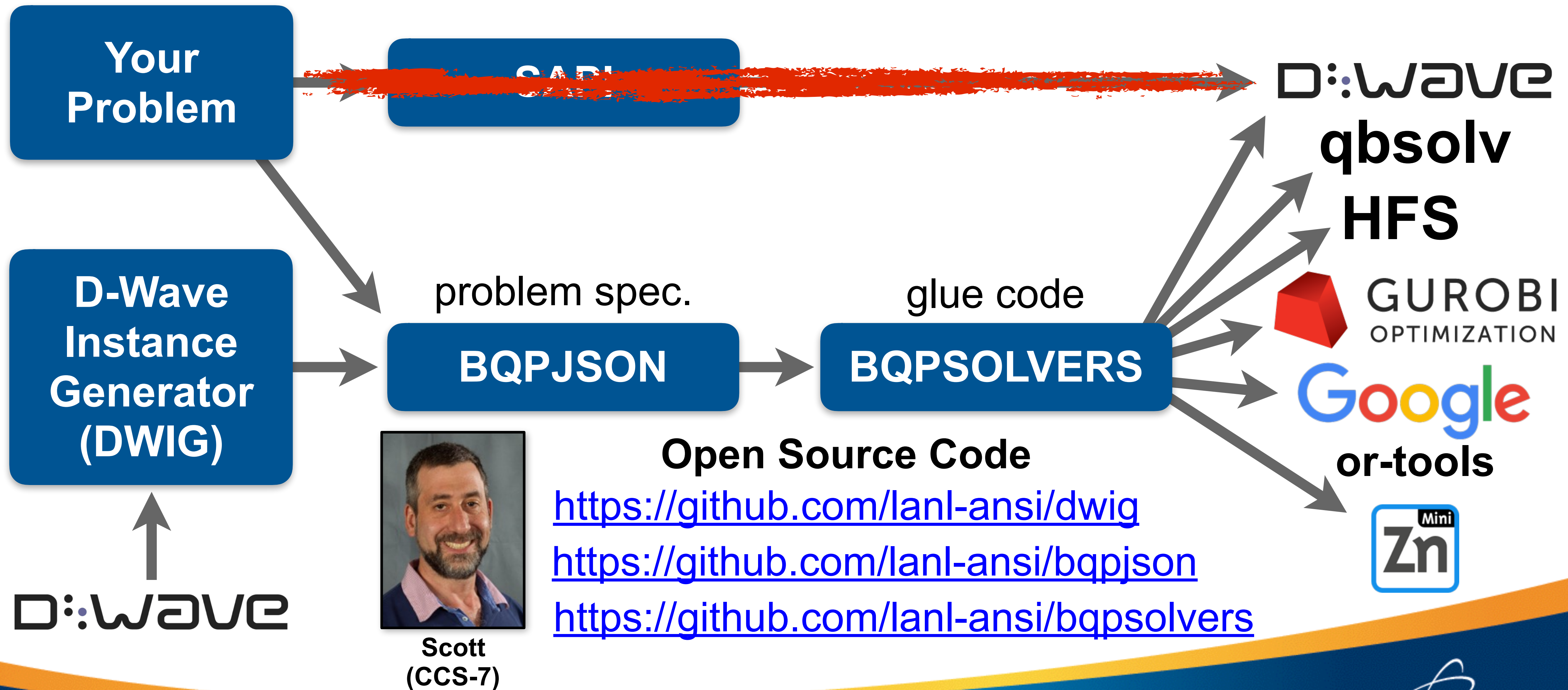


C_{12}

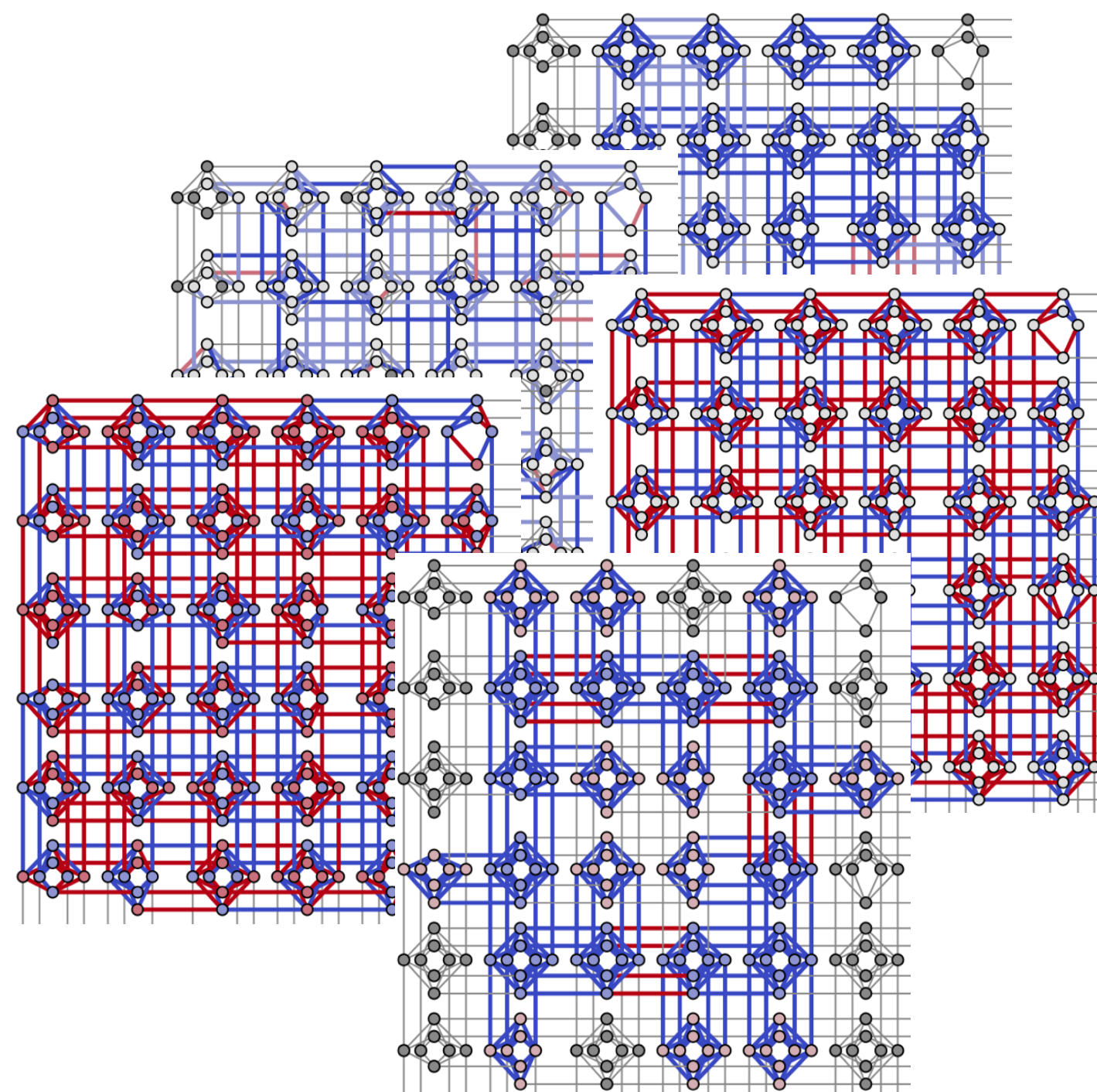
DW2X

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Open-Source B-QP Tools



**D-Wave
Instance
Generator
(DWIG)**



BQPSOLVERS

D-WAVE

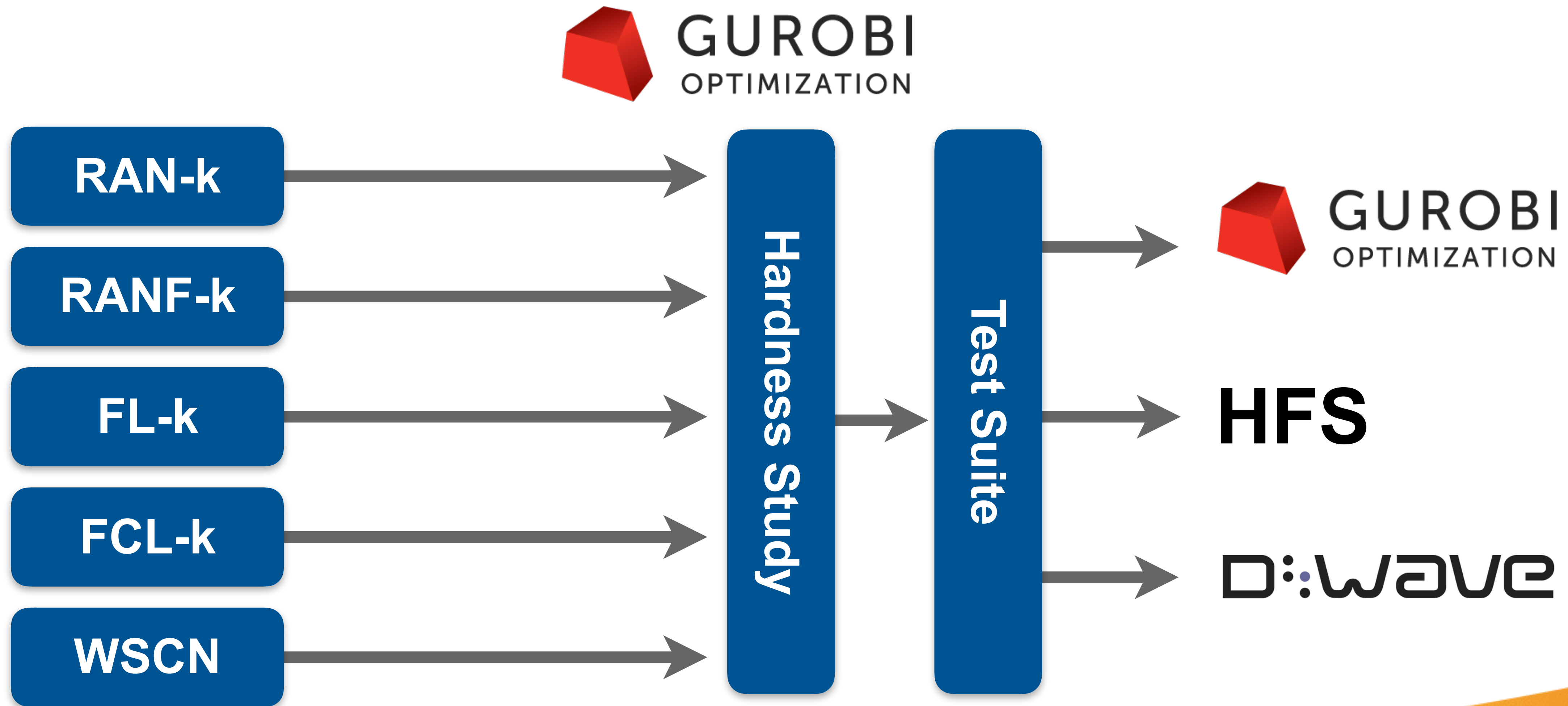
HFS



GUROBI
OPTIMIZATION

Time for the Fun Stuff!

Structure of the Benchmark Study

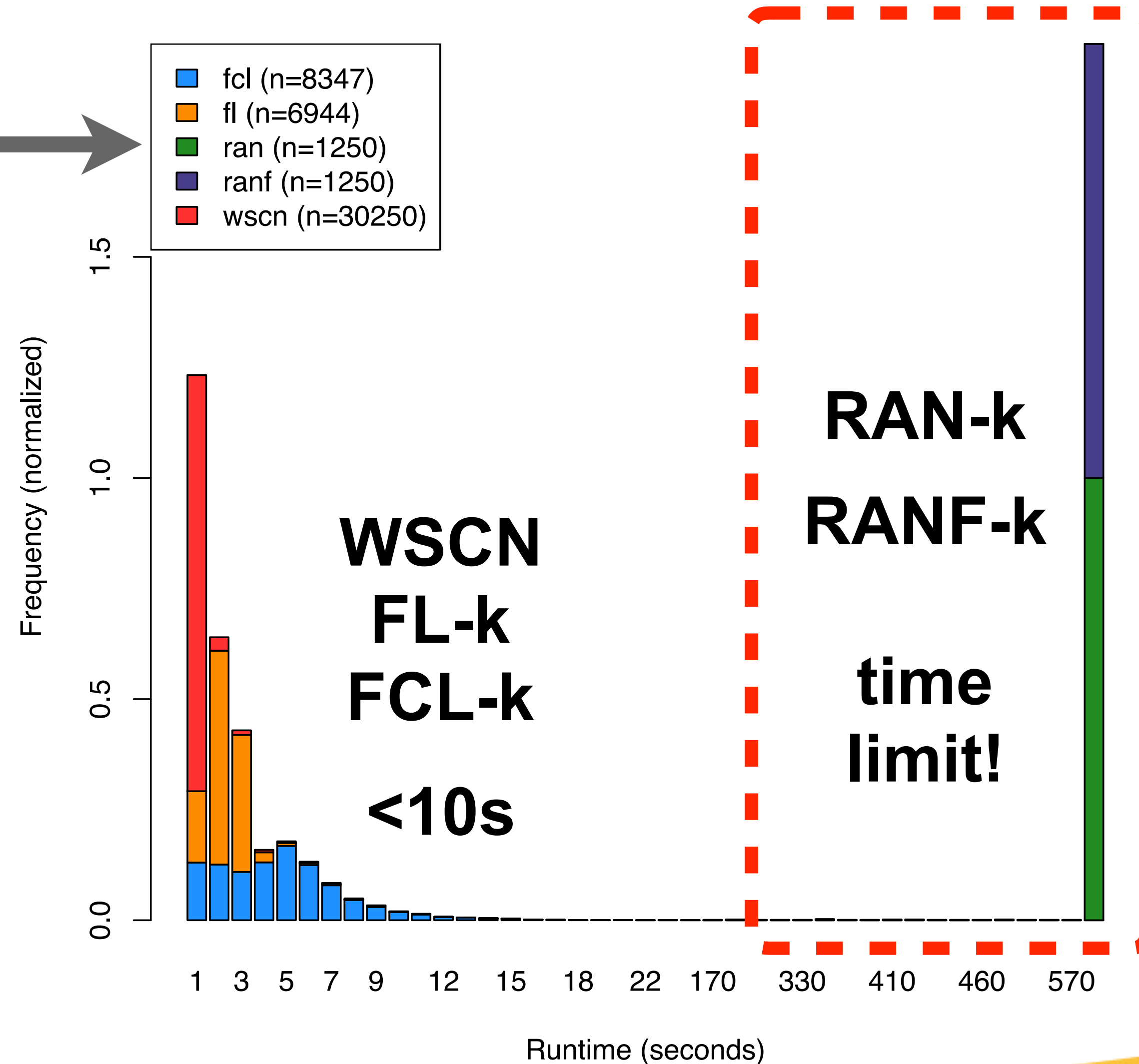


Problem Hardness Test

Runtime Distribution
Color Coded
by Problem

40,000+
Cases

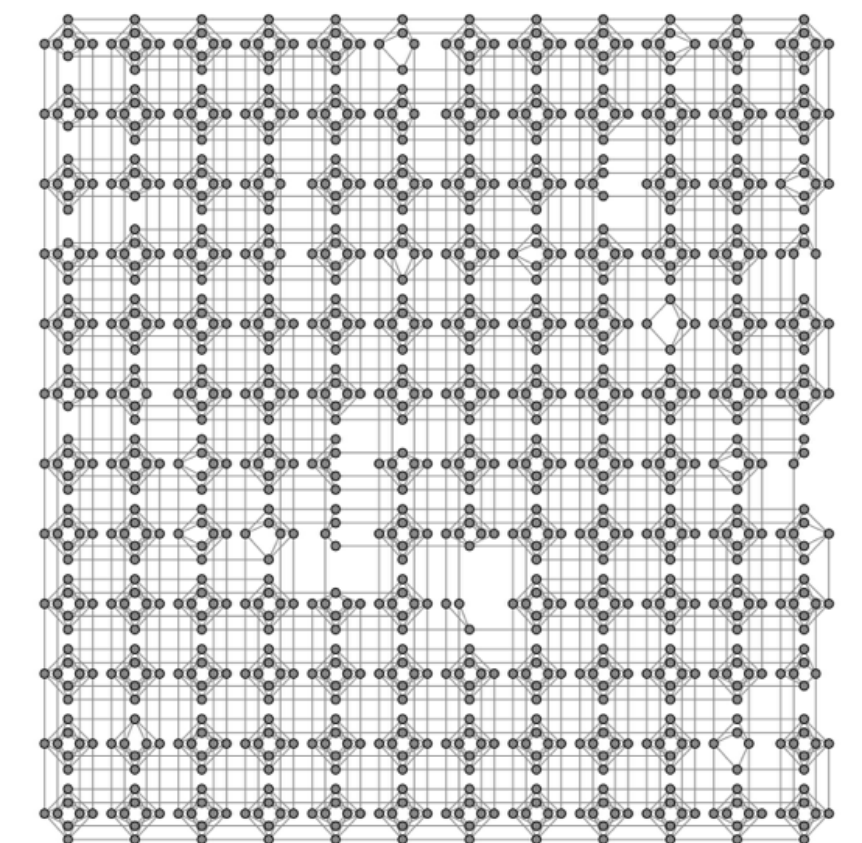
What about the
values of k?



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OPTIMIZATION

600 second
time limit

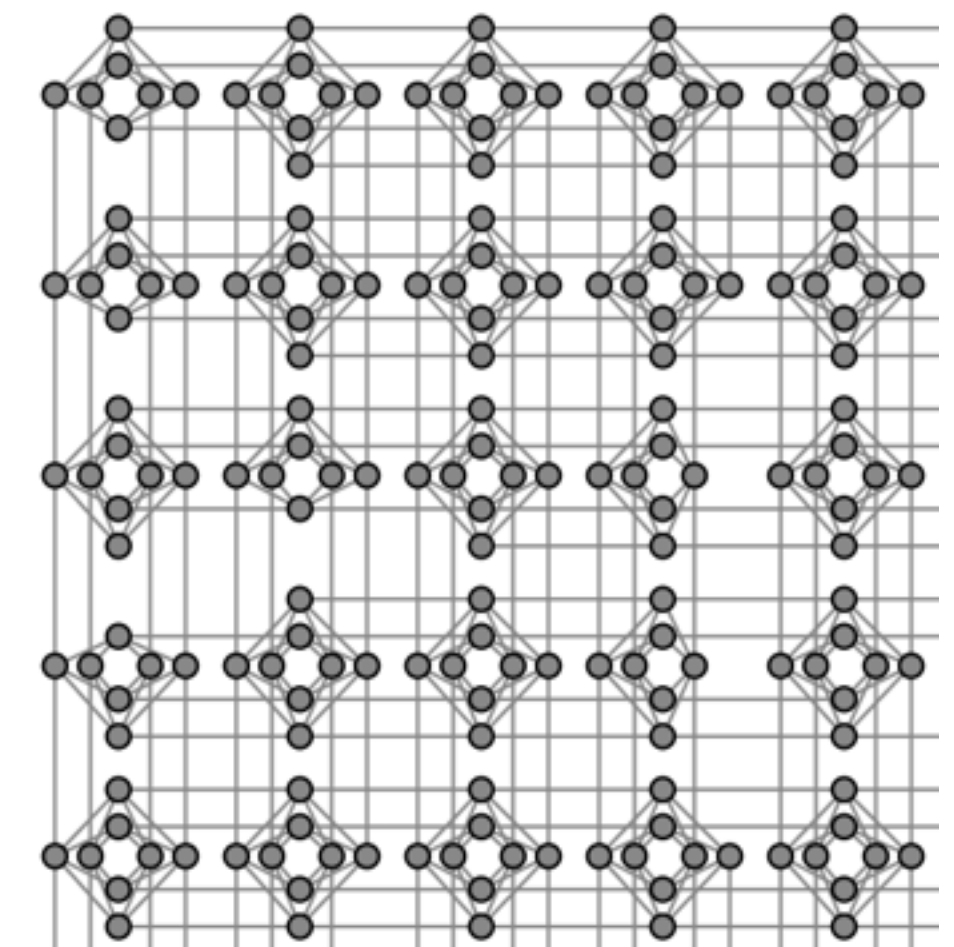
\mathcal{C}_{12} Max Size



How does Hardness change with K?



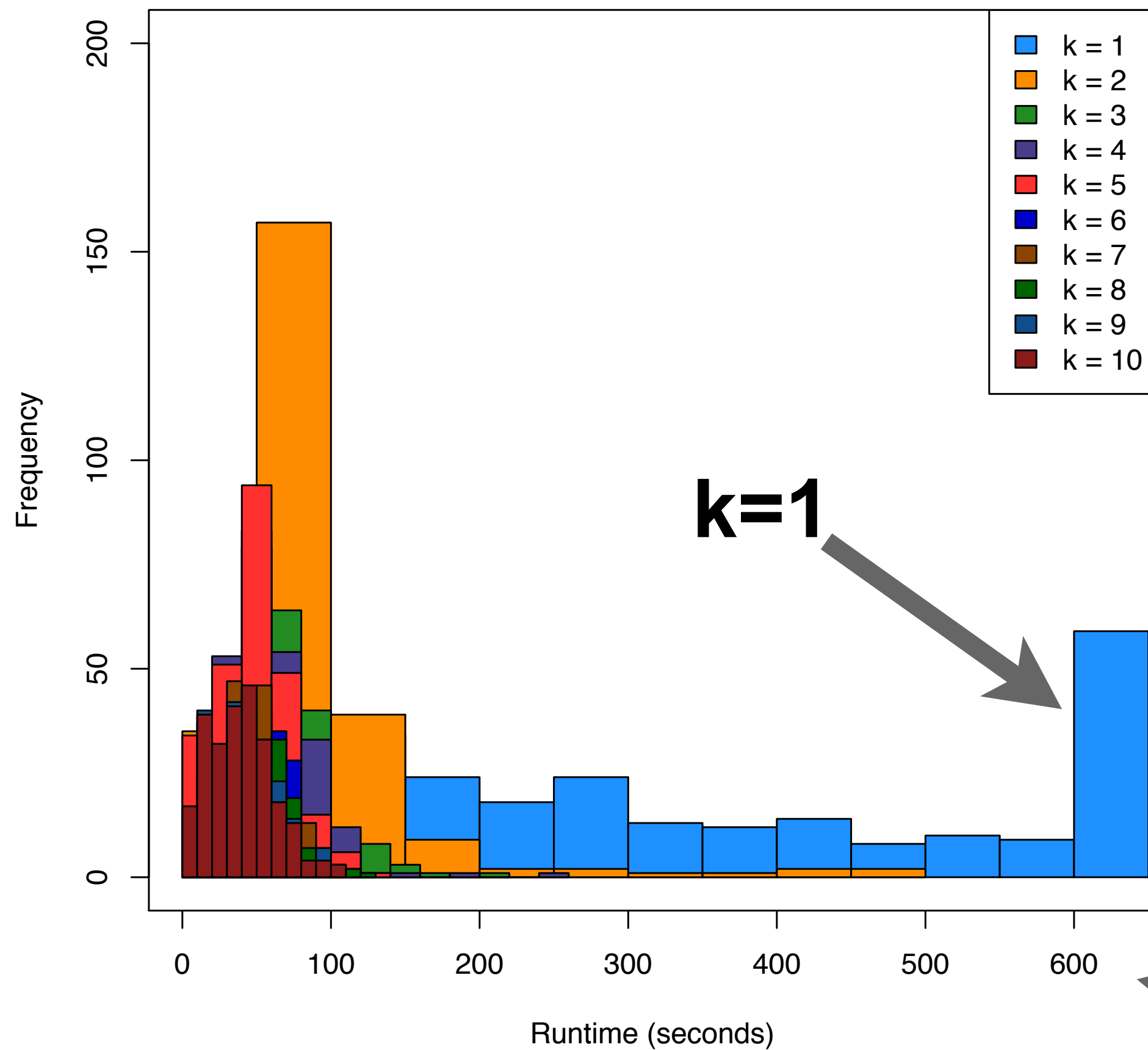
**optimality proof in
600 sec. or less**



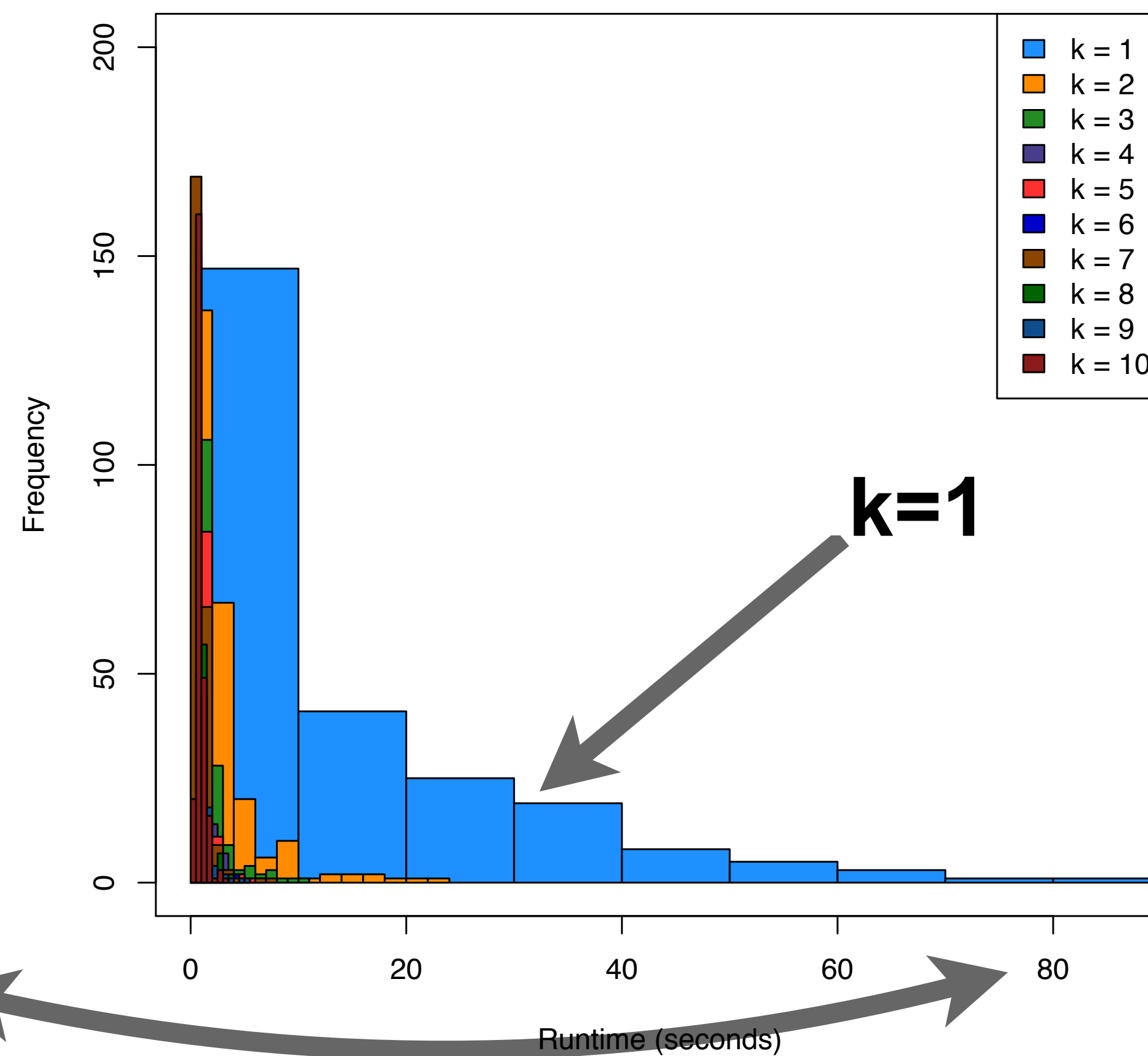
$$\mathcal{C}_5$$

$$2^{194} \approx 10^{58}$$

RAN-k



RANF-k



RAN > RANF

These guys know what they are talking about

“Have you tried setting the couplers to -1,1 at random?”

09/2016



Marc
(T-4)



Andrey
(CNLS/T-4)



Sidhant
(T-5)

Focusing

A Detailed Study of RAN-1



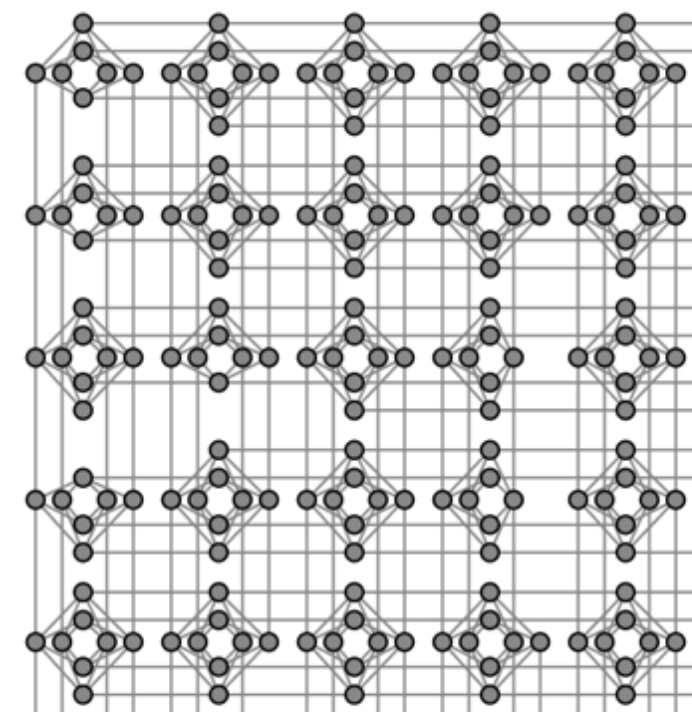
HFS

D:wave

(Similar story for RANF-1, omitted for time)

Detailed Runtime Studies

Part 1



C_5

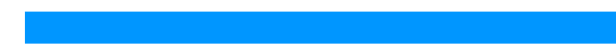
Quality Validation
(Known Global Optimum)



Wishful Extrapolation



GUROBI
OPTIMIZATION



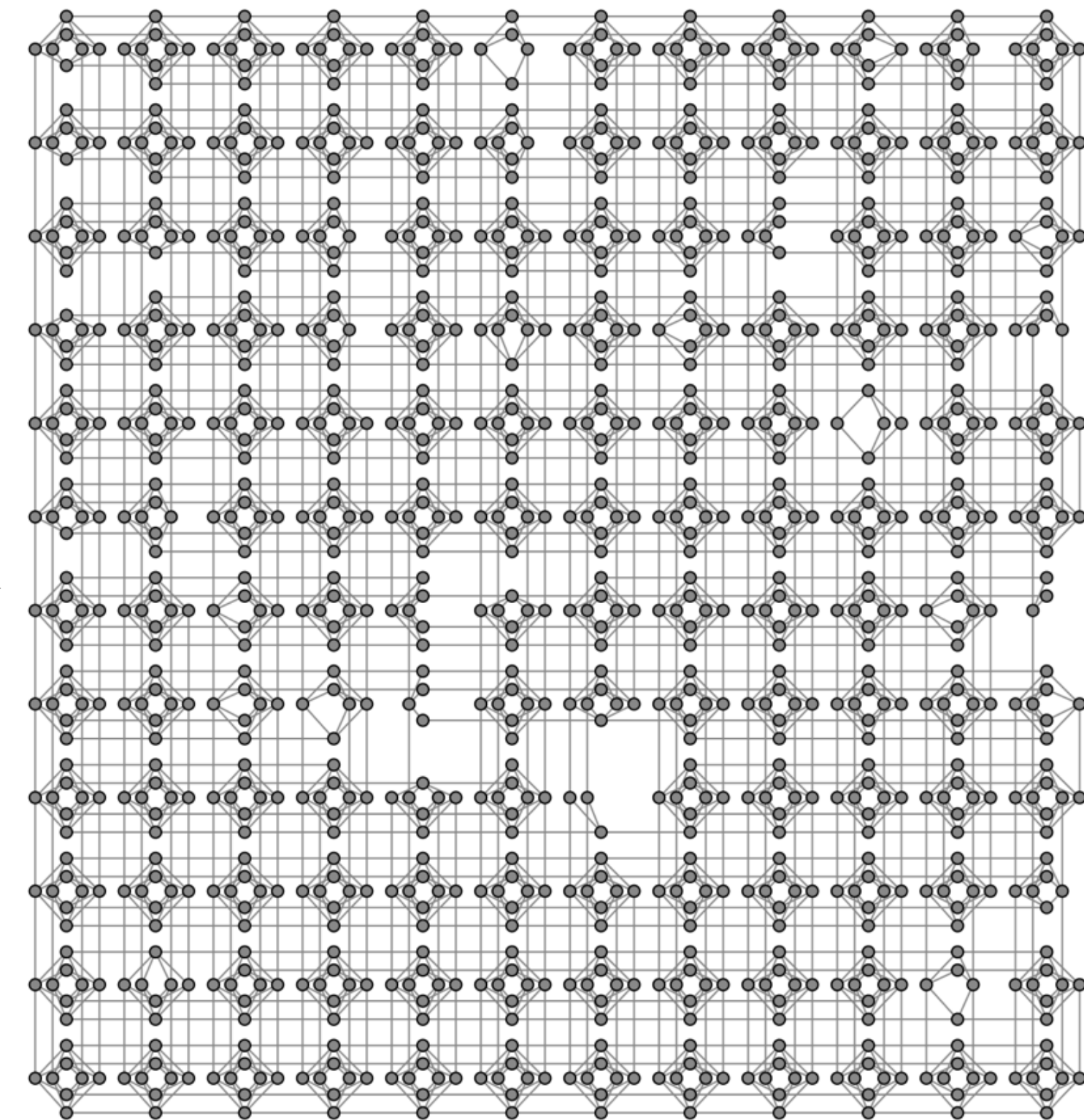
D:wave



HFS

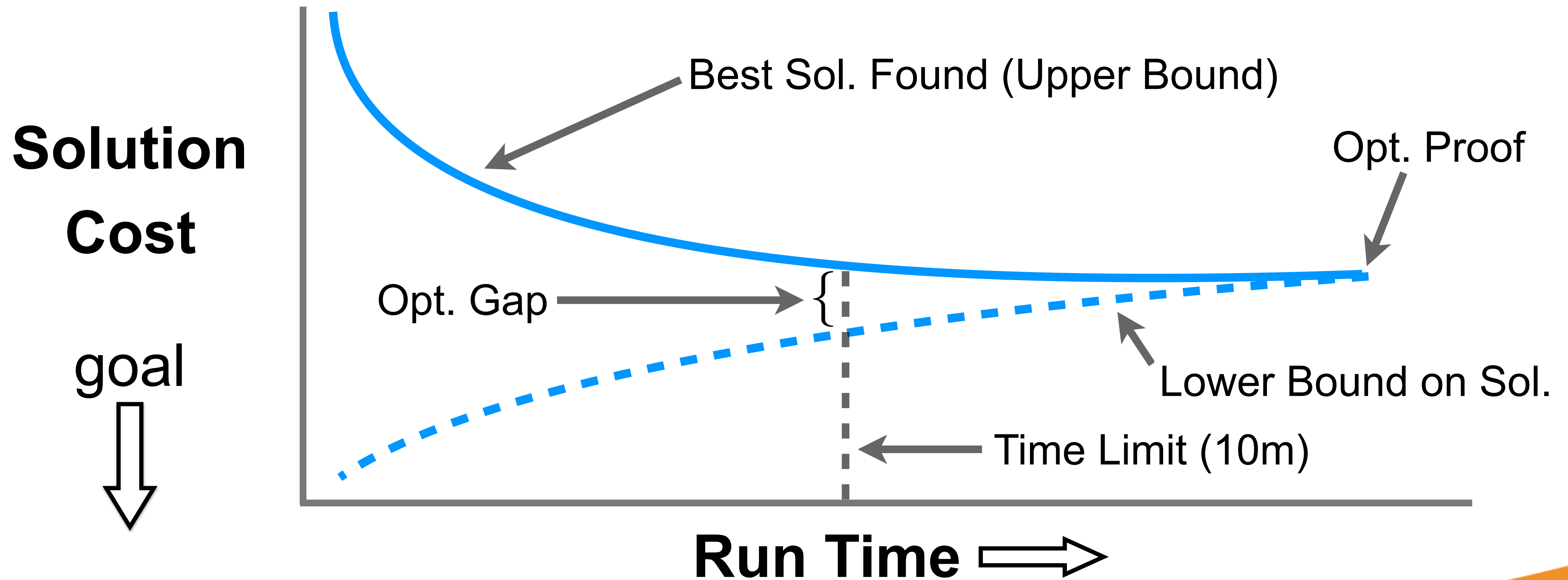


Part 2



C_{12}

Detailed Runtime Studies



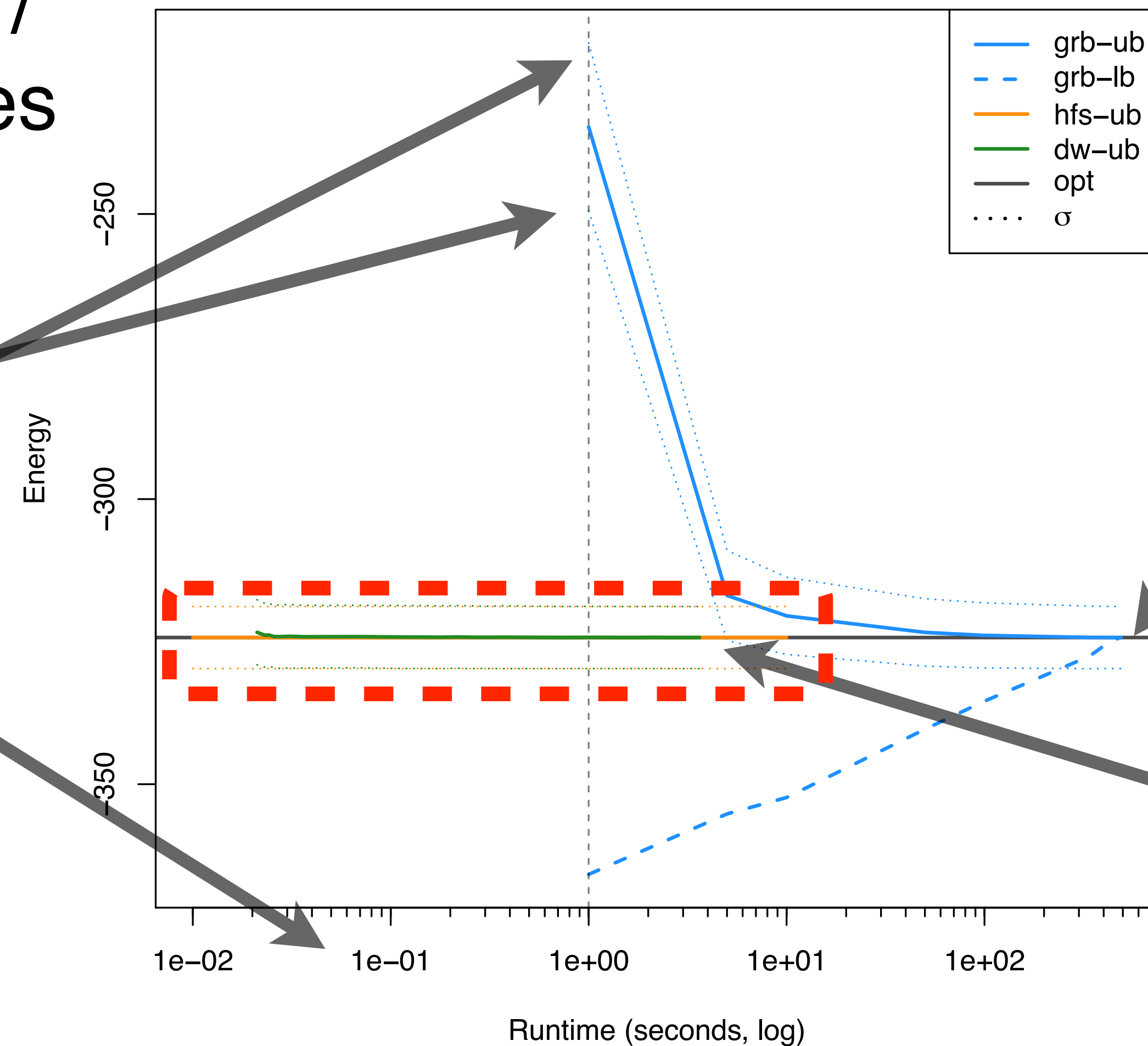
Detailed Runtime Study 1 (RAN-1) \mathcal{C}_5

Average Objective /
Energy of 200 Cases

Variance in Cases
not the algorithm

Logarithmic
runtime scale

RAN-1 Runtime Trend

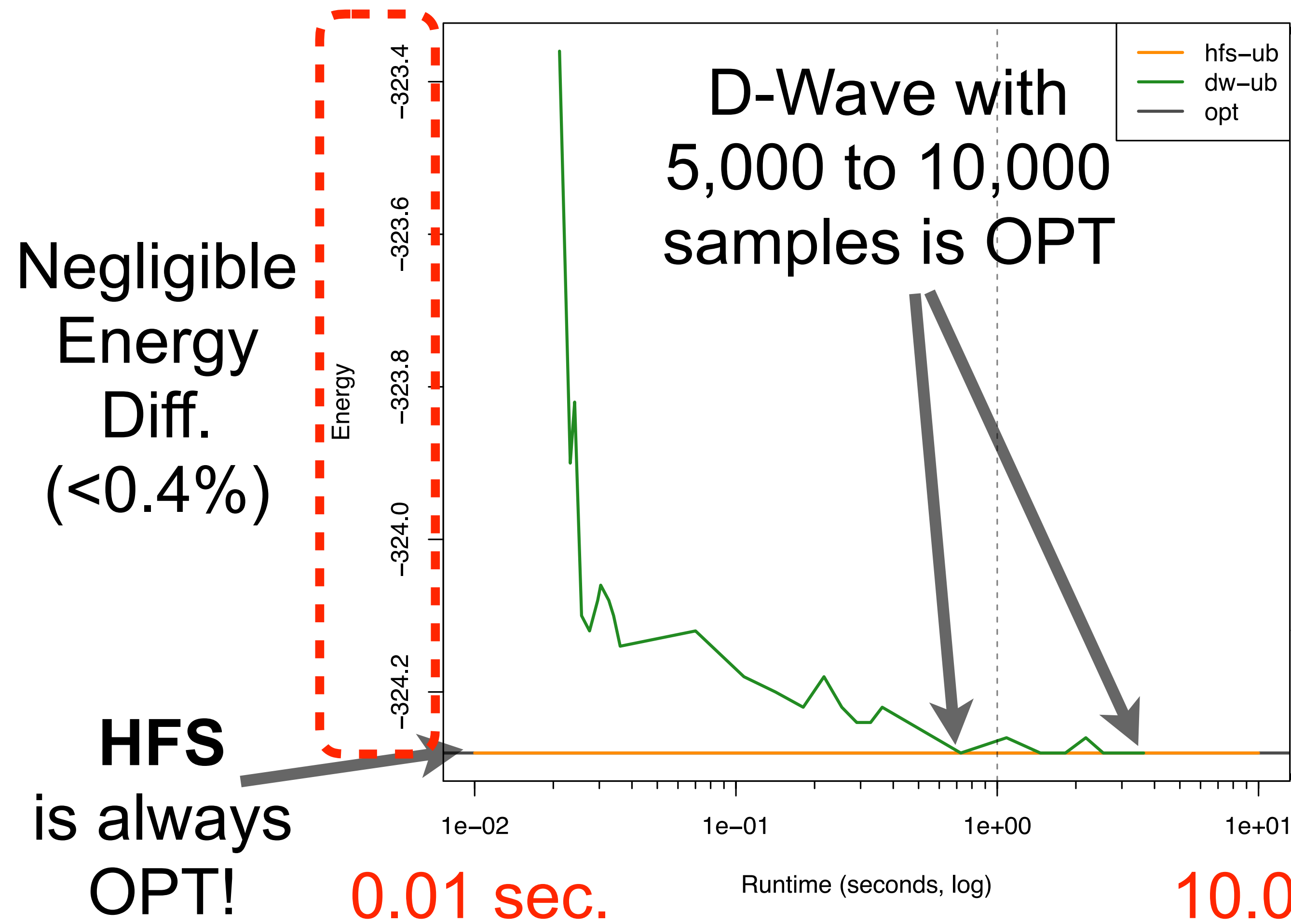


Optimality Proof
for all 200 Cases

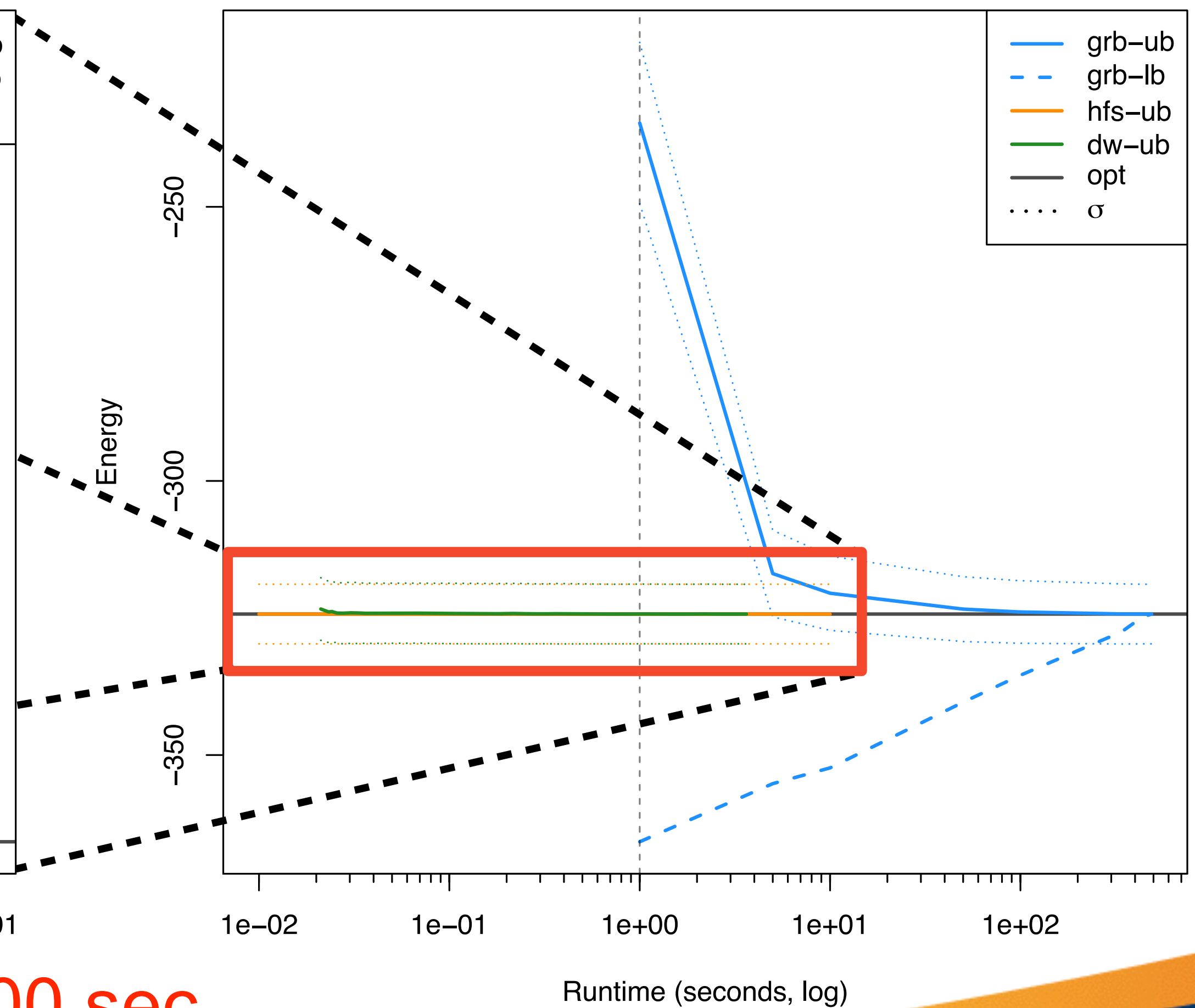
HFS and D-Wave
Indistinguishable

Detailed Runtime Study 1 (RAN-1) \mathcal{C}_5

RAN-1 Runtime Trend



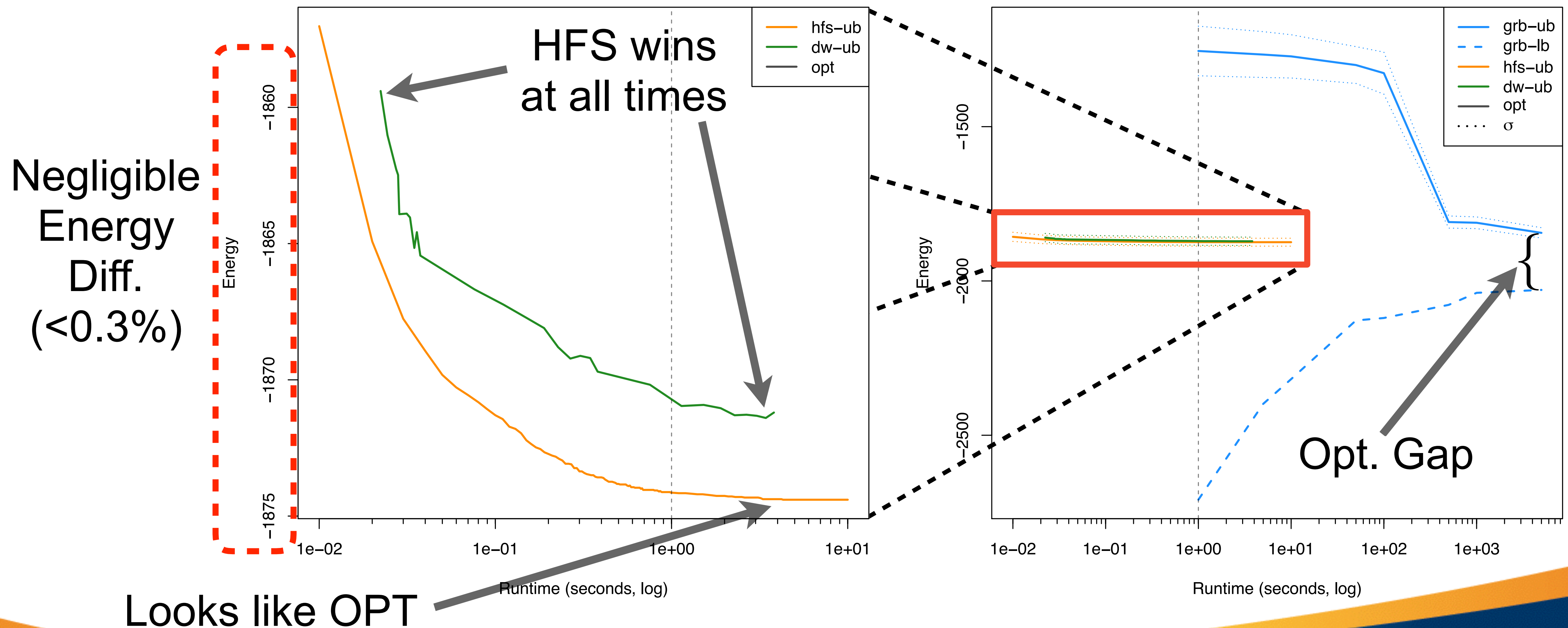
RAN-1 Runtime Trend



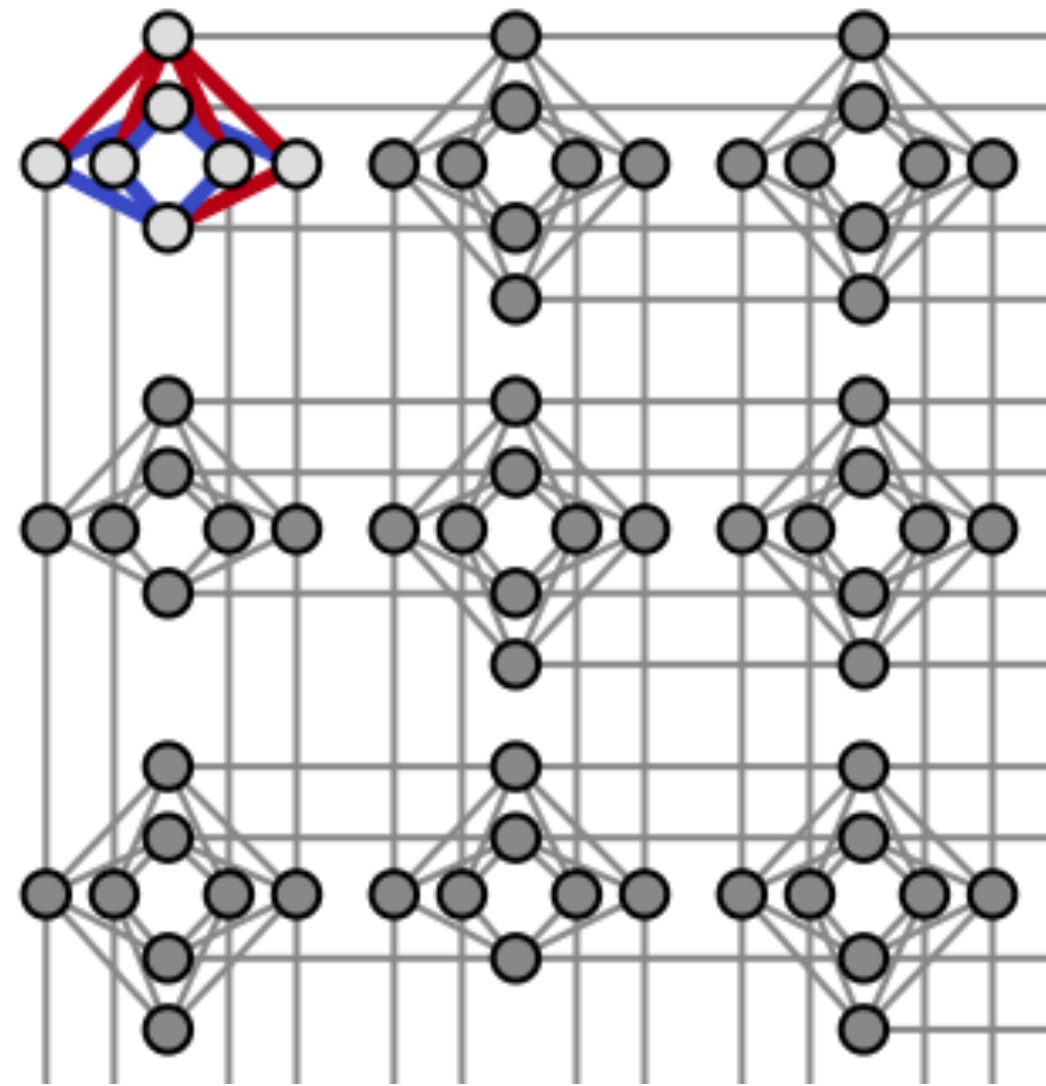
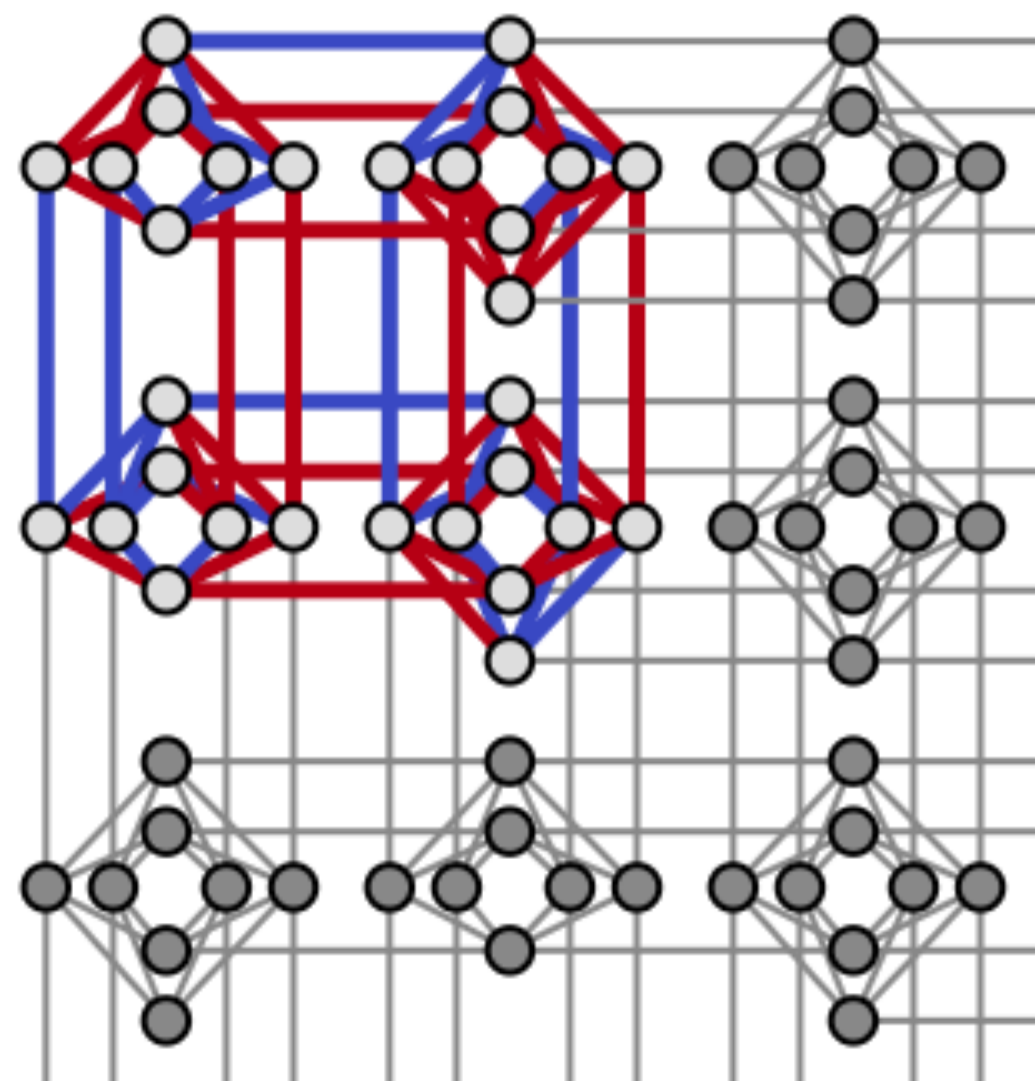
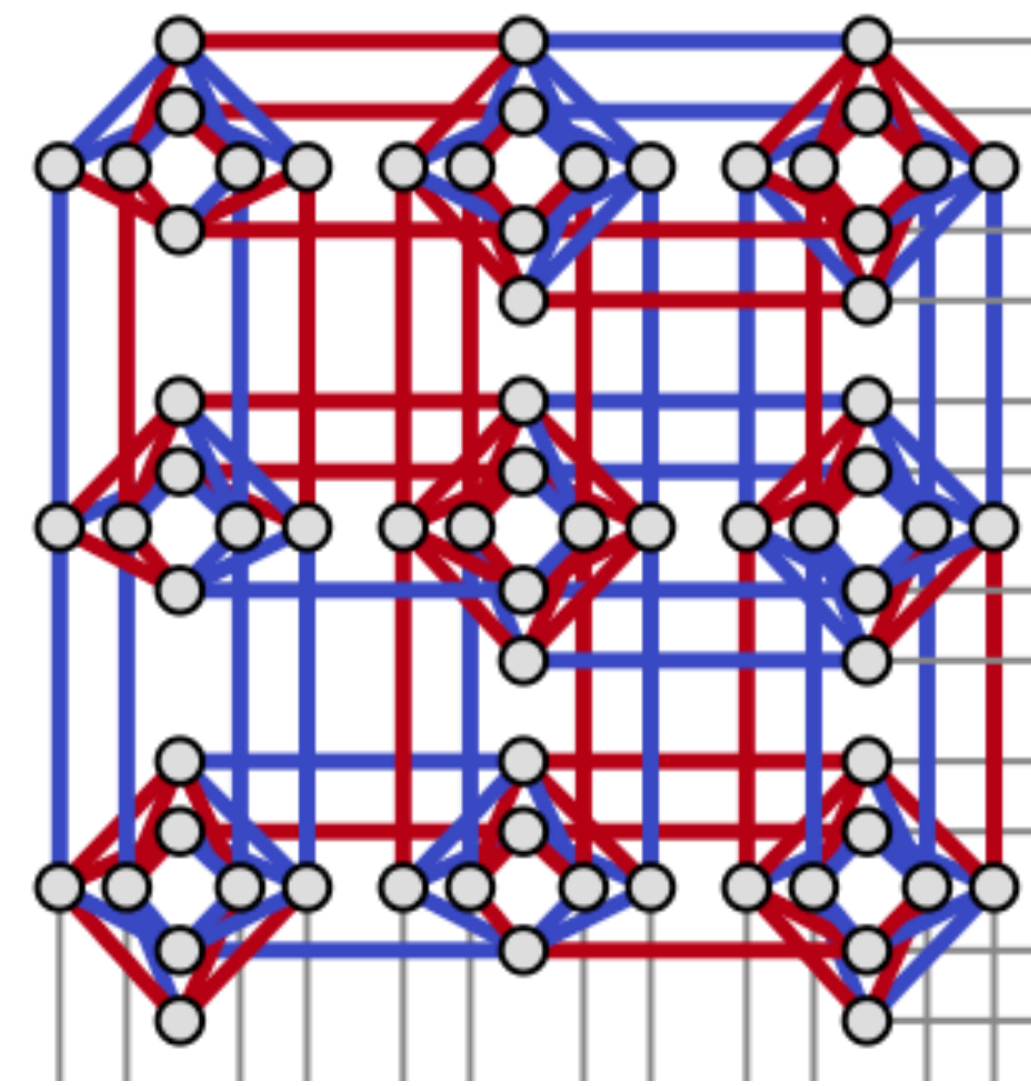
Detailed Runtime Study 2 (RAN-1) \mathcal{C}_{12}

RAN-1 Runtime Trend

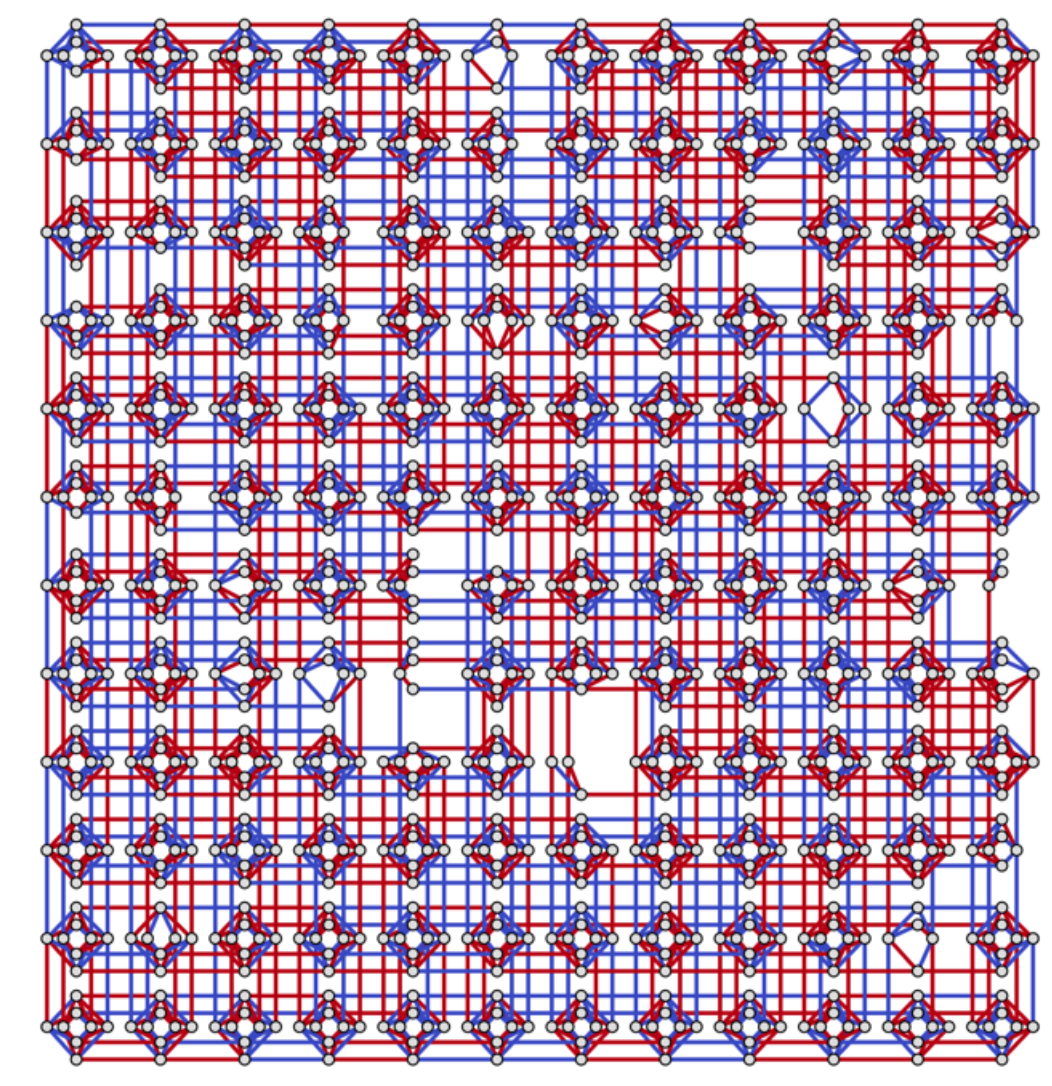
RAN-1 Runtime Trend



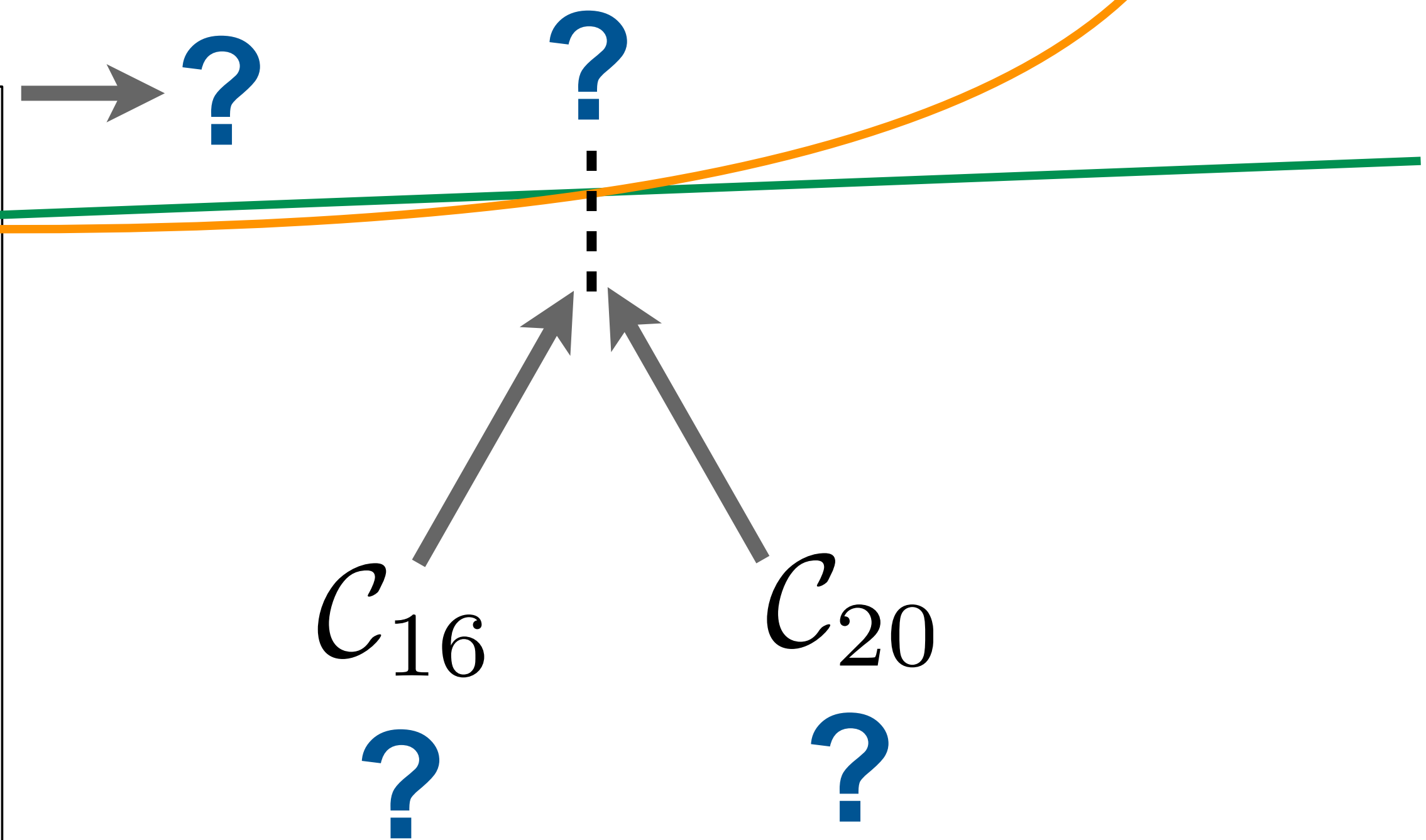
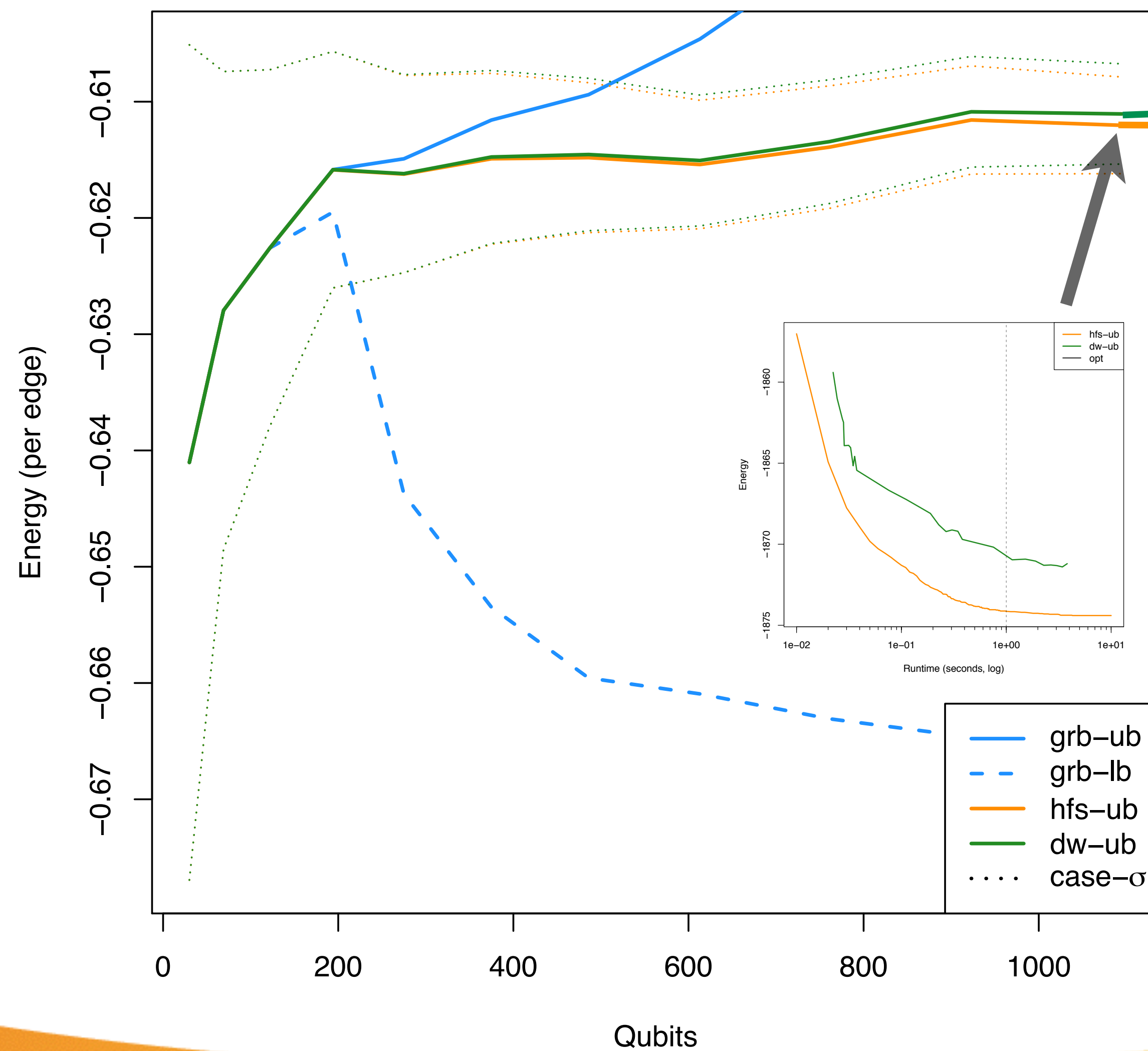
Scalability Study

 \mathcal{C}_1

 \mathcal{C}_2

 \mathcal{C}_3


...

 \mathcal{C}_{12}


Scalability Study RAN-1



A Word of Caution about RAN-1

- The relative difference in energy between the **best** HFS solution and the **worst** HFS solution is only $<1\%$
 - And a similar property is true for the D-Wave!
- This suggests that the RAN problem has many **nearly-equal local minima**
 - This property is not desirable for **benchmarking heuristics** (e.g. SA, TabuSearch, HFS)
- Continued work is needed to design generators of **more challenging test cases!**

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Concluding Thoughts

- It seems that all of the popular test cases from the literature are “**easy**”
- Our D-Wave 2X chip is reliable for well-suited optimization applications (e.g. maxcut), but the point where it will overtake classical heuristics is not yet clear (**2000Q anyone?**)
- We still have **more questions**, than answers!

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Special Thanks



**Ryan
(CCS-7)**

**Admin of ASC's
Darwin Cluster**

Q?

Project Workload

D-Wave Jobs: 60,176

D-Wave CPU Days: 1.16

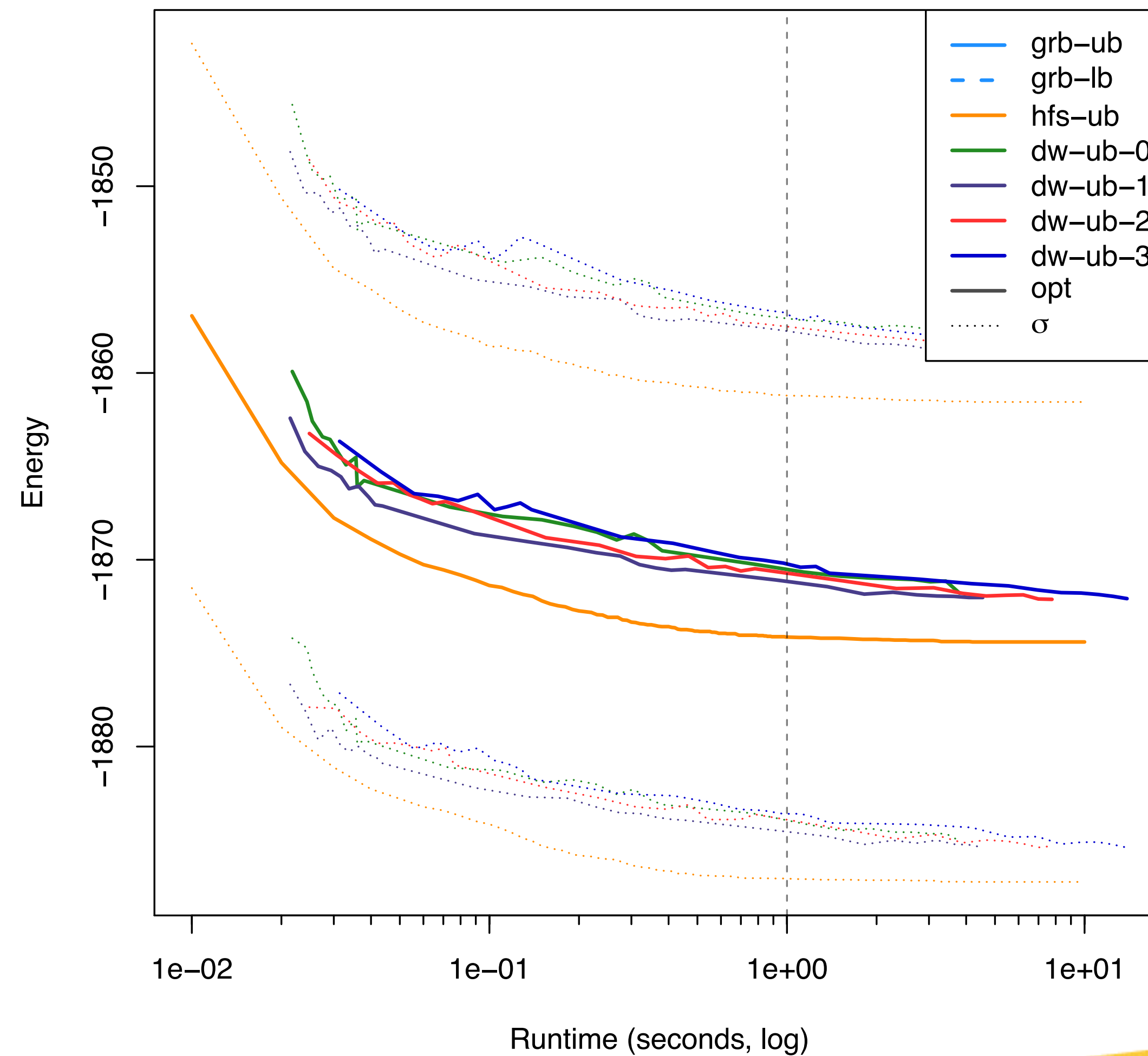
Darwin Jobs: 1,025,122

Darwin CPU Days: 1,290

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Detailed Annealing Time Study \mathcal{C}_{12}

RAN-1 Runtime Trend



0 = 5 microsec.

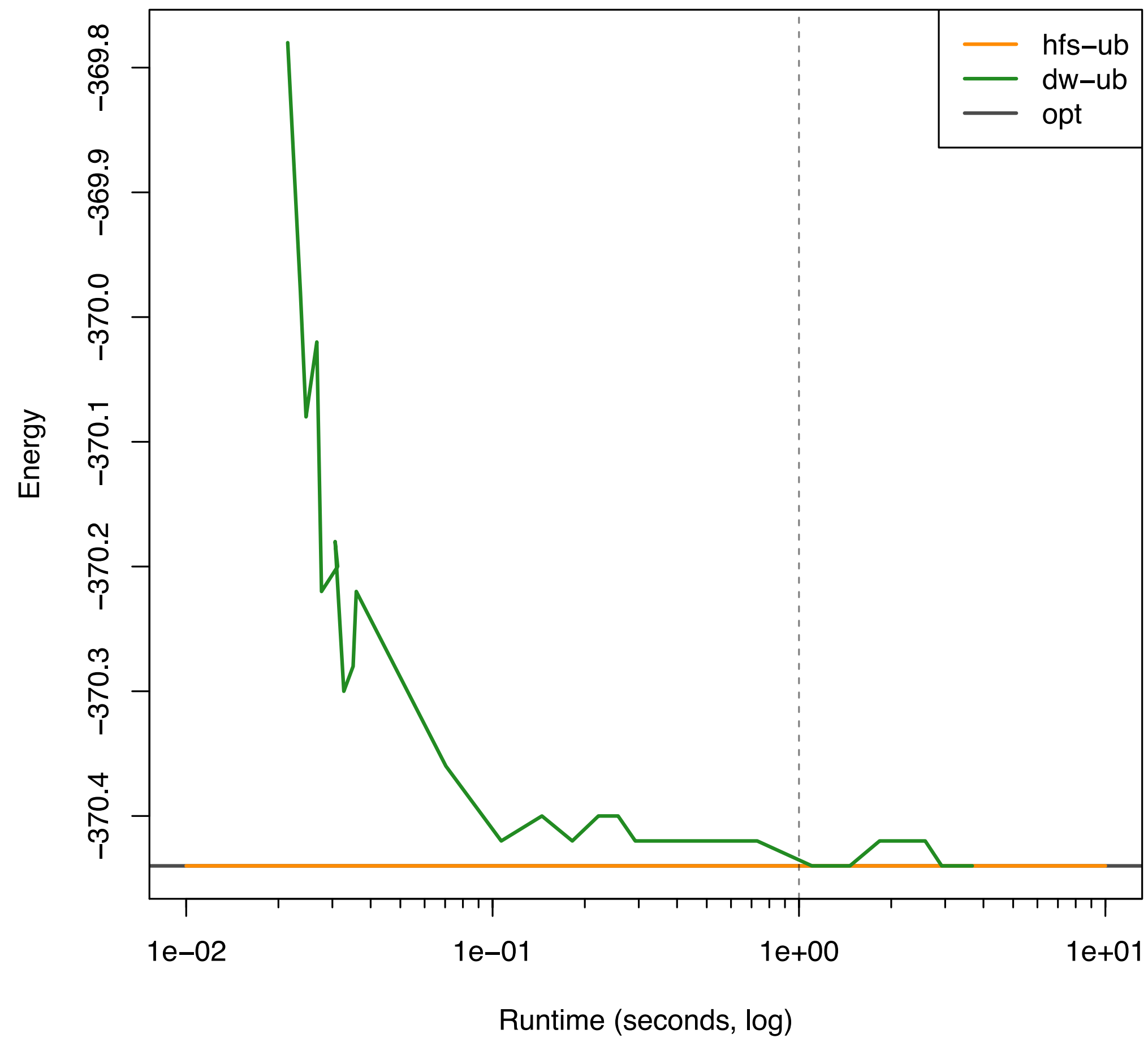
1 = 80 microsec.

2 = 400 microsec.

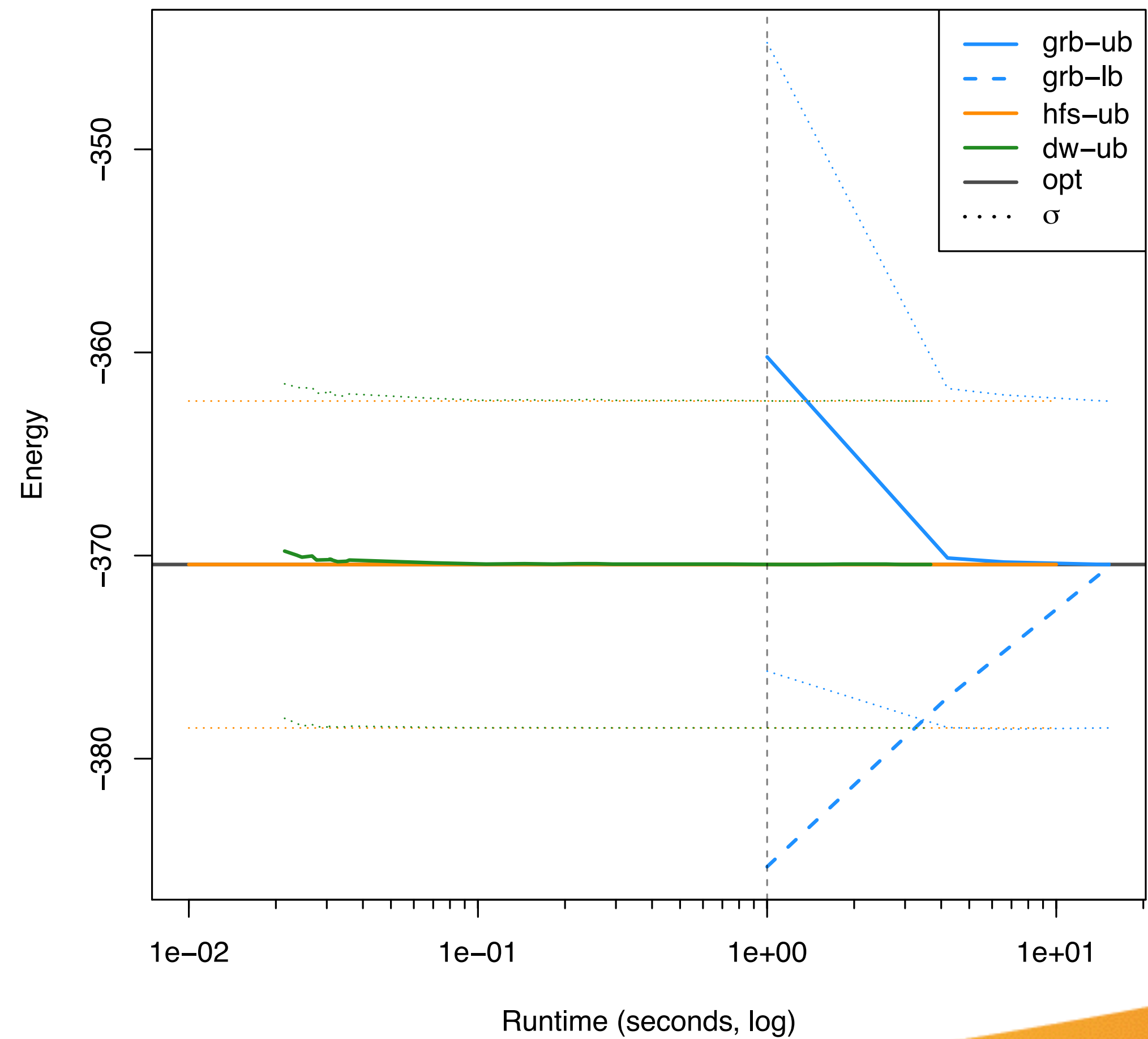
3 = 1000 microsec.

Detailed Runtime Study 1 (RANF-1) \mathcal{C}_5

RANF-1 Runtime Trend

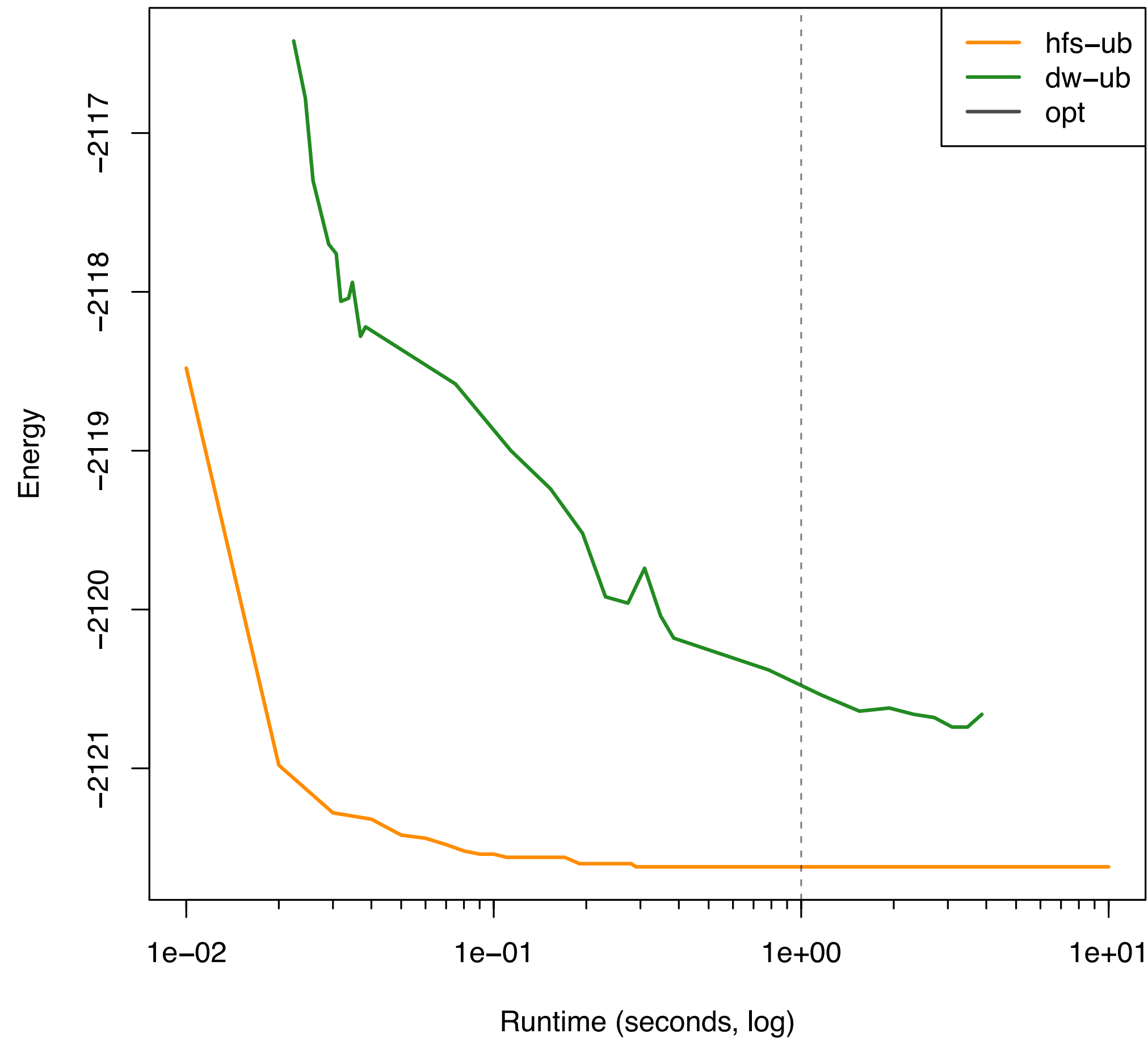


RANF-1 Runtime Trend

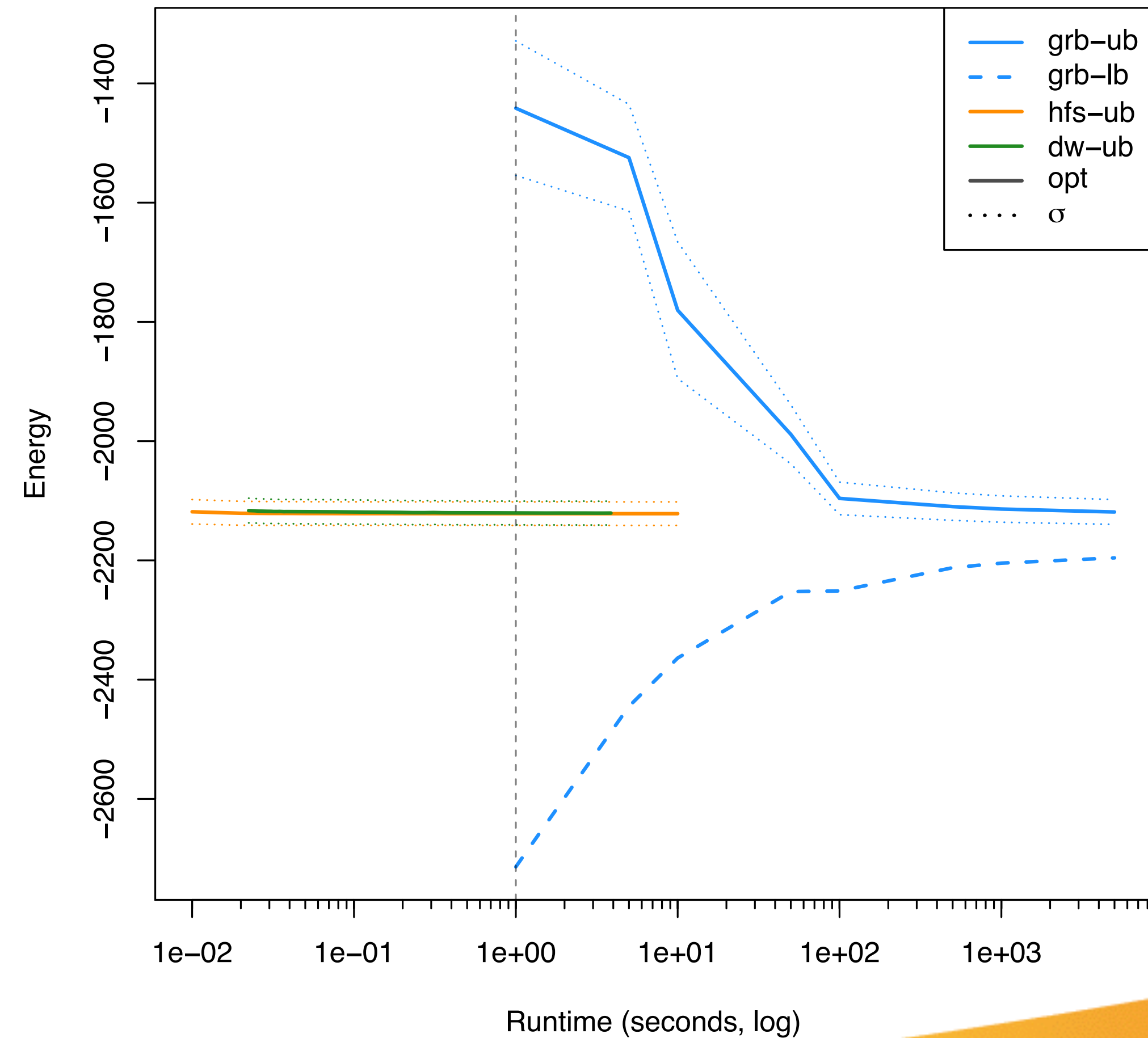


Detailed Runtime Study 2 (RANF-1) \mathcal{C}_{12}

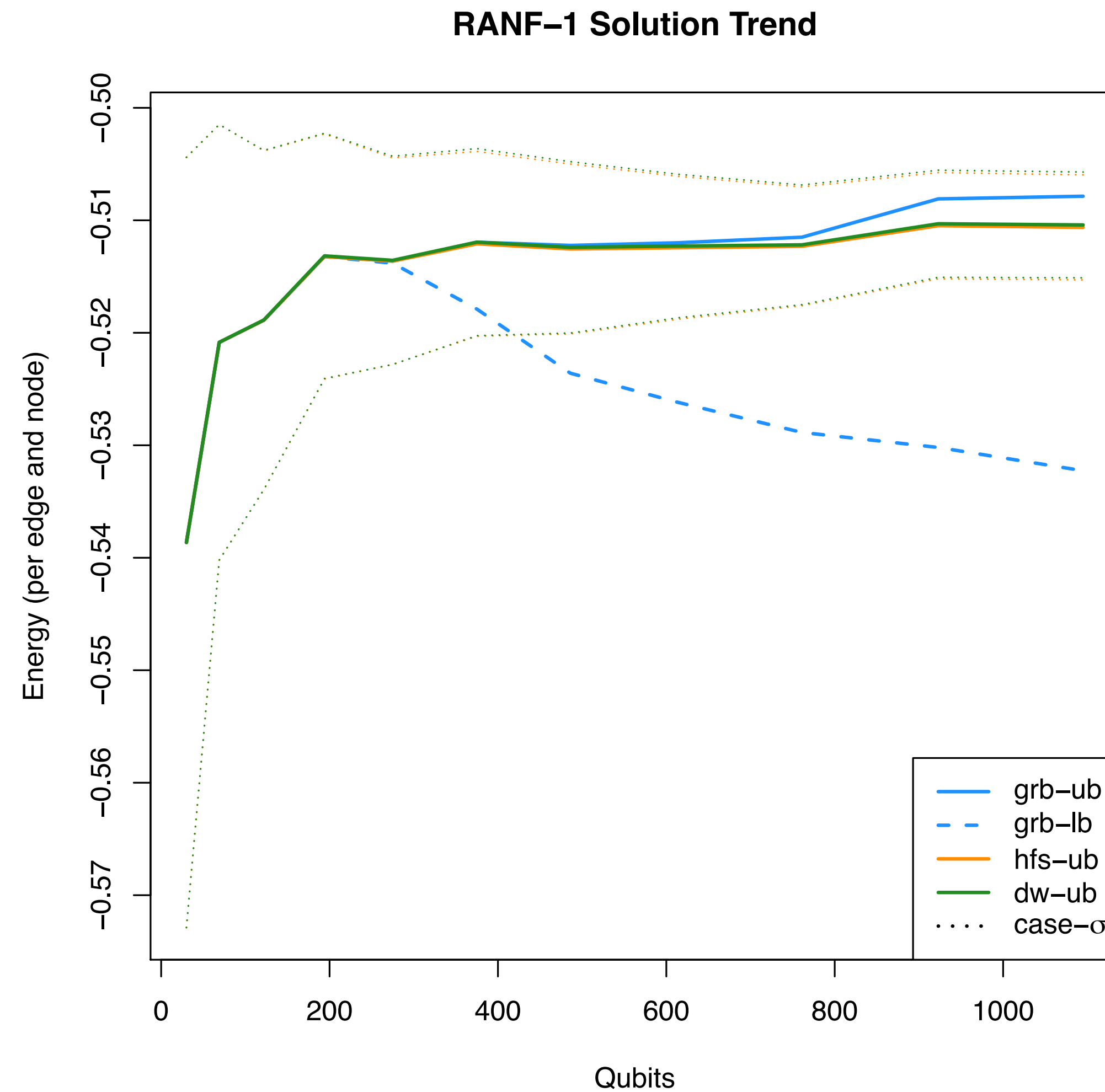
RANF-1 Runtime Trend



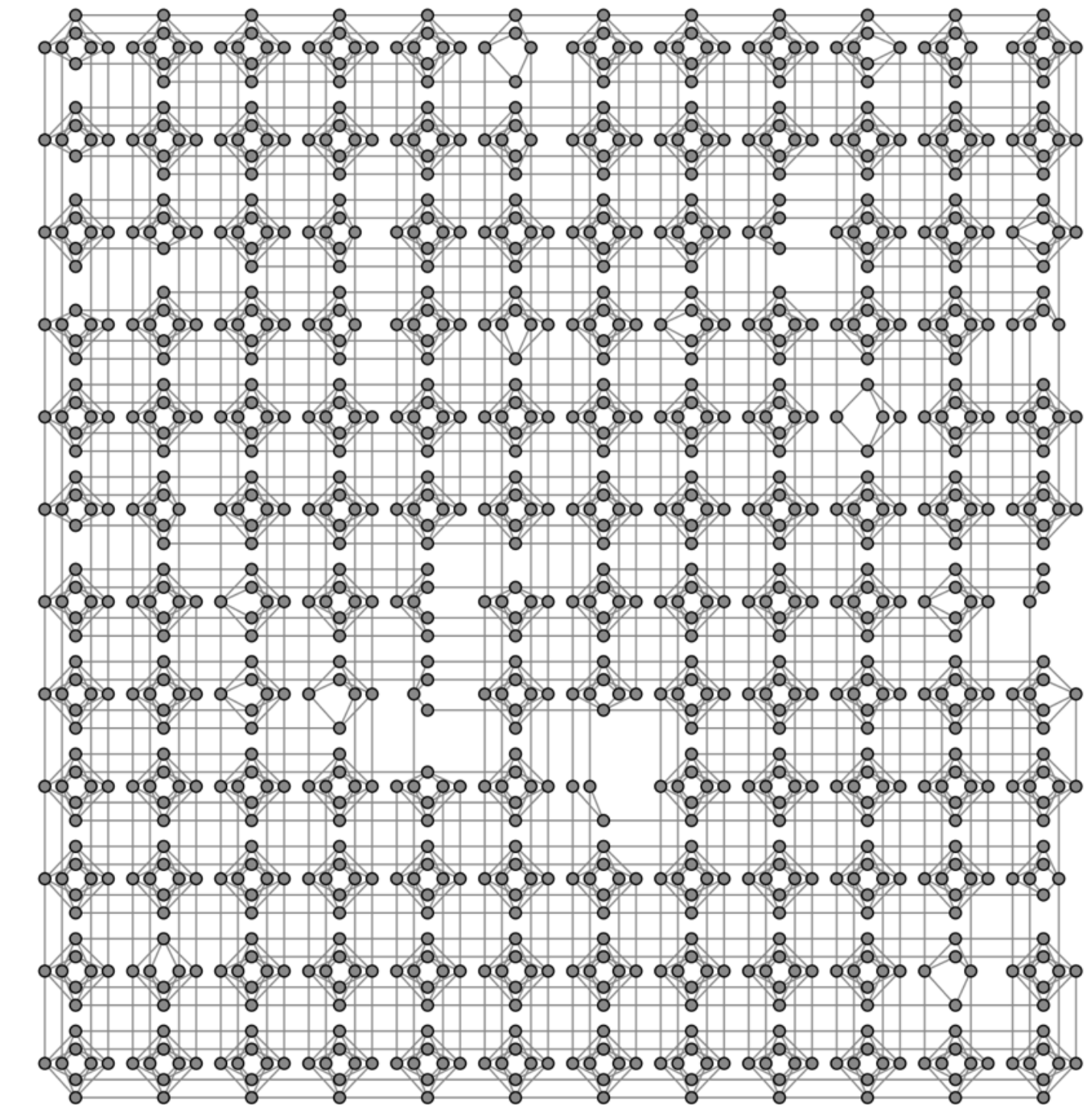
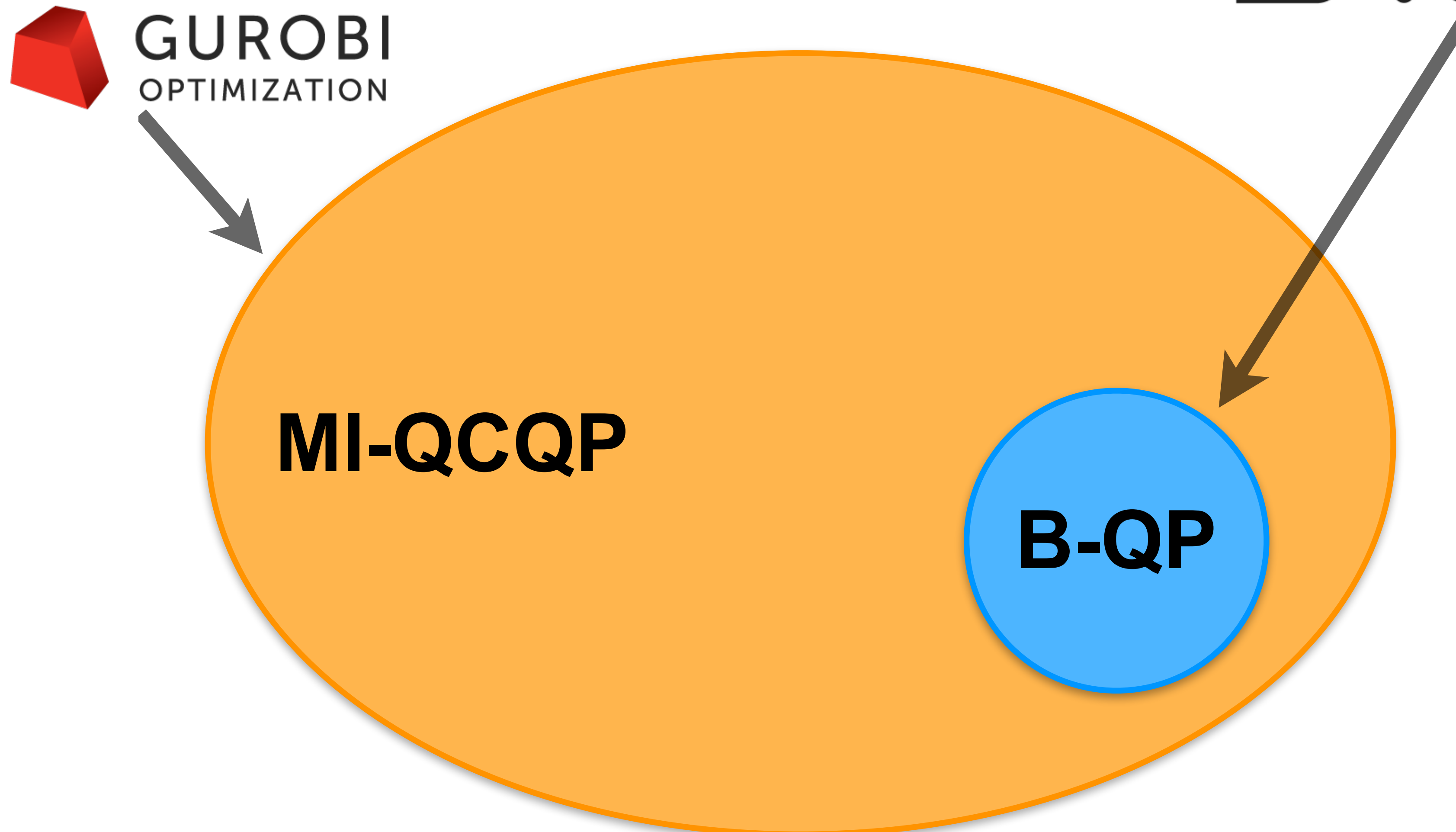
RANF-1 Runtime Trend



Scalability Study RANF-1



Problem Classes


 C_{12}

DW2X

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